Research achievement (研究成果の英訳)

Itsuko Kai

Generalization of flux of viscous conservation law

I have studied the large-time behavior of solutions to viscous conservation law with non-convex flux. In the mathematical model, there are many examples of viscous conservation law which has non-convex flux. For example, "Two phase flow problem, Etching of semiconductor and Night traffic flow problem" has non-convex flux. In 1997, Matsumura-Mei proved the asymptotic stability of viscous shock profile for a system of visco-elasticity with a non-convex nonlinearity. On the other hand, there were no research results for viscous conservation law with non-convex flux. We showed in paper [1] that the asymptotic stability of superposition of stationary solution and rarefaction wave by using not only energy method which was introduced by Matsumura-Mei but also the new weight function defined by us. We also showed the existence of solution in the better functional space than that of previous results.

We also derived the convergence rate of solution to the superposition of stationary solution and rarefaction wave in the paper [2]. It was shown not only the case where the flux is convex but also the case where the flux is in wider class. For the proof, we employ the weight function which is defined in [1], and combined L^p-L^1-estimate.

Asymptotic behavior for Damped-wave equation

We considered the large-time behavior of the solution for Damped wave equation where the convection term is not convex in [3] and showed the asymptotic stability of the superposition of stationary solution and small rarefaction wave. The fact that the sub-characteristic condition is enough to be supposed on the far field was also found out in [3] though this condition had been assumed on whole space in the previous researches. We also derived the convergence rate of the solution to the stationary waves. For this proof, a new weight function was introduced.

In [4], we extend the result [3] and showed that the asymptotic stability of non-degenerate stationary solution. Anti-derivative method and weighted energy method was used for the proof. We also derive convergence rate by using space-time weighted energy method. As in paper [3], we also showed that the sub-characteristic condition is enough to be imposed only on the far field.

Radial symmetric solution of Burgers equation

We study the large-time behavior of the radial symmetric solution for Burgers equation on the outer space of multi dimensional space, where the corresponding Riemann problem for the hyperbolic part admits the rarefaction wave in [5]. It is known by the work of Liu-Matsumura-Nishihara('98) that the asymptotic states of solution of scalar viscous conservation law are divided into three cases dependent on the signs of the characteristic speeds $f'(u_{\pm})$ of the boundary state and the far field state. In the present research, it is proved that even for the radial symmetric solution for the Burgers equation, the asymptotic states are similar to the one of viscous conservation law. Furthermore, we also derived the time convergence rate. The proof is given by a standard L^2 energy method.