Results of research

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Papers are referred by numbers in "List of my papers"

I have been studying "the knots contained in spatial embeddings of graphs". If there exists a cycle of a spatial graph which is equivalent to a knot K, then the knot K is contained in the spatial graph. In 1983, Conway and Gordon proved that every spatial embedding of the complete graph with 7 vertices contains the nontrivial knot. The purpose of my research is to study the generalization of Conway-Gordon's result.

A linear embedding of a graph is an embedding which maps each edge to a single straight line segment. A linear embedding such that each vertex of K_n is mapped into a spiral is called by a column embedding. In the joint paper [1] with T. Tanaka, we proved that "every column embedding of the complete graph with $(2n-1)$ vertices or $2n$ vertices contains the torus knot of type $(2n-5,2)$". For column embeddings, we gave a sufficient condition for knots to be contained in it of the complete graph.

A circular embedding of a graph is defined as an embedding such that there exists an arc contained in the image of any edge. For any graph, there exist a circular embedding representing it. For a particular spatial graph, there exists an embedding which is not equivalent to it. From here, we focus on knots, which are considered as knots, which are considered as spatial graphs homeomorphic to the circle. For a knot K, the circular number $Circ(K)$ of K is defined by the minimal number of the number of vertices a circular embedding of a graph equivalent to K. In the case of linear embeddings, the number of edges consisting a nontrivial knot is at least 6. In the case of circular embeddings, the number of edges consisting a nontrivial knot is at least 3. It seems that a circular embeddings are more suitable than linear embeddings for the study of generic embeddings. In the joint paper [1] with Dr. Tanaka, we obtained inequalities of the circular number of K and some invariants such as a stick number, an arc index, a crossing number, a bridge number and a superbridge number. Moreover, we prove the following results. "A knot K is a trefoil knot if and only if $\mathrm{Circ}(K)$ is equal to 3. " "If K is the figure-eight knot, then $\mathrm{Circ}(K)$ is equal to 4. " "If K is the connected sum of a trefoil knot and its mirror image, then $\mathrm{Circ}(K)$ is equal to 4. " In general, the circular number is not additive under connected sum. Finally, we defined another invariant of a knot associated with a circular embedding. Let K be a knot. We define $u_n(K)$ by the minimal number of crossing changes such that K is transformed into a circular embedding consisting of n round arcs. We obtain the following result. "For a trefoil knot K, $u_n(K)$ is 1 if $n \leq 2$ and 0 if $n \geq 3$. " Note that if $n \leq 2$ then $u_n(K)$ is exactly equal to the unknotting number of K. So, we may say that $u_n(K)$ is a generalization of the unknotting number.

I also have been studying "the generalization of an enumeration on self-complementary graphs for edge colored graphs". It was known, as Royle's conjecture, that the number of self-complementary graphs is equal to the difference of the numbers of isomorphic classes having even edges and odd edges. Nakamoto, Shirakura and Tazawa proved Royle's conjecture in 2008. Moreover, Tazawa and Ueno generalize the proof of Royle's conjecture for bipartite graphs in 2010. Any graph is understood as a 2-coloerd complete graph. So, we can consider the similar problem for r-colored complete graphs. By this point of view, we define cyclic automorphism as a generalization of self-complementarity. For edge colored bipartite graphs and digraphs, we generalize essential formula of Royle's conjecture. Although Tazawa and Ueno proved their result by using Burnside's Lemma. The lemma is not suitable for our case. We obtain our results by generalizing Burnside's Lemma. By using our results, we can enumerate the number of cyclic automorphic graphs.

In general, a graph is defined as a pair of sets of vertices and edges, and an edge is a segment(set of 2 vertices) which adjacents two vertices. As a generalization of the notion, the hyper-graph whose edges are regarded as surfaces (set of 3 vertices) or solids (set of 4 vertices) is known. We generalize our results stated above to the cases of hyper-graphs. Here, we treated hyper-graphs whose edges are sets of h vertices. By this restriction, our results are naturally generalized to the case of hyper-graphs.