Research statement (project)

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Over the next few years I am planning to work about the following projects.

To unify toric manifolds and toric HK manifolds from topological point of view. Toric HK manifolds themselves are quite different from toric manifolds as a space. However, similar phenomena often occurs between them: for example, their equivariant cohomology rings are isomorphic to the Stanley-Reisner rings of their corresponding combinatorial objects; or equivariant cohomological rigidity holds for both of them. Under the motivation of "to unify toric and toric HK manifolds from topological point of view", I am studying some class which may be regarded as topological generalization of toric HK manifolds in (30), Using the result in (15) I get the equivariant cohomology rings of them. This study may be regarded as the geometric counterpart of the study of hypertorus graphs. Over the next few years, I would like to study them from both of geometry and topology more deeply, and I aim to find the quaternionic analogue of torus manifolds.

Extended actions of GKM manifolds. According to Wiemeler, in general, if torus manifolds have extended actions, then their transformation groups are SU or SO-types, i.e., groups whose root systems are type A_ℓ, B_ℓ or D_ℓ, such as (6), (9). So we are naturally led to ask the problem "what kind of classes of manifolds have the extended actions with other types?" GKM manifolds could be one of such classes. Because all of the homogeneous spaces G/H with same ranks are contained in GKM manifolds, all root systems of type A_ℓ–G_2 can be appeared as the extended actions on GKM manifolds.

Now, I am studying extended actions of GKM manifolds with Masuda-Wiemeler via the combinatorial structure of GKM graphs.

Moreover, unlike torus manifolds, we can consider the extension of torus actions to the higher dimensional torus actions in GKM manifolds. I am also studying about the obstruction of this extension with Park.

Classification problems for several classes. I will study the next topic related to rigidity and classification problems.

(1) Cohomological rigidity problem of (quasi)toric manifolds;
(2) Characterization of torus manifolds which satisfy cohomological rigidity;
(3) Classification of simply connected torus manifolds (study some conjecture);
(4) Homotopical rigidity problem of $\mathbb{C}P$-towers;
(5) Characterize when $\mathbb{C}P$-tower satisfies cohomological rigidity or rigidity by other cohomology theories (I and Ray are working on progress for KO theory);
(6) Cohomological rigidity problem of toric HK manifolds with fixed dimension;
(7) Cohomological rigidity problem of aspherical small covers.

In particular, the third problem is related to the open problem "characterization of multi-fans induced from torus manifolds". So, I believe the third problem is one of the most important problem in toric topology.

Relations with other areas via GKM graph. Recently, GKM graph is used in many areas (not only topology). For example, in mathematical physics, FTCY graph is defined to compute the Gromov-Witten invariant of toric Calabi-Yau 3-folds (the FTCY graph can be regarded as the special type of GKM graphs). In representation theory (algebraic geometry), Fiebig translated the Lustig's conjecture into the problem on GKM graphs and proved it. I believe that there must be many possibilities to connect between topology and other areas (in particular combinatorics) by using GKM graphs. Although I still have to learn other areas, I assume I can contribute to find connections between seemingly unconnected fields of mathematics (by GKM graphs). In order to achieve that, I would like to discuss with mathematicians who study other areas, and share our ideas.

I am looking forward to discussing with other mathematicians in OCAMI and having an opportunity to contribute to the development of the institute.