Research plans

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It is an interesting problem to observe the moduli space of n-noids of genus one ($n \in \mathbf{N}$). However, this problem is very difficult because of the period problem with respect to the homology basis of the torus. Therefore we consider n-end catenoids, which have a symmetry with respect to a plane and all of whose ends are on the symmetric plane. For such surfaces, we present three problems:

1. Collapse of n-end catenoids

R. Schoen proved that there does not exist a catenoid with handles. Hence, if the flux data of an n-end catenoid of genus one goes near to that of "the catenoid of genus one", then the surface must collapse. It is well known that a subsequence of arbitrary n-end catenoids converges to a union of (branched) minimal surfaces with finite total curvature. We construct new examples of families whose flux data go near to that of "the catenoid of genus one". We would like to estimate the flux of the handles of such surfaces, and observe the concrete collapse.

2. n-end catenoids with $D_n \times \mathbf{Z}_2$ symmetry (D_n: the dihedral group)

Here we consider n-end catenoids with $D_n \times \mathbf{Z}_2$ symmetry. Then the angles of neighboring ends must be $360^\circ \times k/n$ ($k \in \mathbf{N}$, k,n: mutually disjoint, $k \leq n-1$).

In the case that $n = 2$, by the result of R. Schoen there does not exist a 2-end catenoid.

In the case that $n = 3$, Berglund-Rossman constructed a 3-end catenoid, whose neighboring ends are at an angle of 120°. On the other hand, we prove that there does not exist a 3-end catenoid whose neighboring ends are at an angle of 240°.

We have many other examples in general. Combining these results, we expect a necessary and sufficient condition for the existence of an n-end catenoid with $D_n \times \mathbf{Z}_2$ symmetry as follows: *There exists an n-end catenoid with $D_n \times \mathbf{Z}_2$ symmetry if and only if the angle of neighboring ends is less than* 180°. We would like to verify the conjecture and observe the moduli space of such surfaces.

3. Collapse of n-end catenoid fences

Here we consider n-end catenoid fences, which are singly periodic. If an n-end catenoid fence with $D_n \times \mathbf{Z}_2$ symmetry collapses, then the ratio of the period and the weight converges to $2\pi/\sin(\pi/n)$. We would like to clarify the meaning of the value, and observe the collapse of general n-end catenoid fences.