Research results

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An n-noid is a complete minimal surface in $\mathbf{R}^3$ with finite total curvature and n ends, and all of whose ends are embedded. We can define weights for embedded ends. An embedded end is called a planar (resp. catenoidal) end if the weight of the end is zero (resp. is not zero). In particular, an n-end catenoid is an n-noid with n catenoidal ends.

I have studied n-end catenoids of genus one.

Let X be an n-noid, $\{q_1, \ldots, q_n\}$ the ends of X, w_j the weight of q_j, and G the extended Gauss map of X. Then the pair of sets $\{w_1, \ldots, w_n\}$, $\{G(q_1), \ldots, G(q_n)\}$ is called the flux data of X. By the Gauss' divergence formula, we have $\sum_{j=1}^{n} w_j G(q_j) = 0$. Conversely, we can consider a problem of finding an n-end catenoid that realizes given data w_j and $G(q_j)$ satisfying $\sum_{j=1}^{n} w_j G(q_j) = 0$. Such a problem is called an inverse problem of the flux. In the case of genus zero, S. Kato (OCU), M. Umehara (TITECH) and K. Yamada (TITECH) proved that for almost all flux data, there exists an n-end catenoid realizing the given data.

In the case of genus one, a domain of an n-noid is a punctured torus, therefore the Weierstrass data of an n-noid is expressed by elliptic functions. We observe zeros and poles of elliptic functions, and conclude that n-noids are classified to two classes $\omega = \omega_2$ and $\omega = 0$. Here we describe our research results for each class.

1. The class $\omega = \omega_2$

We give a formulation of n-noids in this class by using functions on annular domains and describe a necessary and sufficient condition for the existence of an n-noid (cf. [1]). The well-known examples with catenoidal ends, such as Costa's example and Berglund-Rossman's example, are in this class. We construct a family of $2N$-end catenoids with $D_N \times \mathbf{Z}_2$ symmetry (D_N : a dihedral group, $N \geq 3$), a family of 4-end catenoids with symmetry of rectangles, and two families of 3-end catenoids with symmetry of isosceles triangles.

2. The class $\omega = 0$

We give a formulation of n-noids in this class by using the Weierstrass zeta function on parallelograms and describe a necessary and sufficient condition for the existence of an n-noid (cf. [2]). In this class, the known examples are Costa's 4-noid and its generalization by Kusner-Schmitt. However, we did not know an example with catenoidal ends. I prove that n-noids of type $\omega = \omega_2$ can be expressed in this class by using their covering space. We construct $2N$-end catenoids with $D_N \times \mathbf{Z}_2$ symmetry ($N \geq 3$, N odd) and N-end catenoids with $D_N \times \mathbf{Z}_2$ symmetry ($N \geq 5$, N odd). We also prove that there does not exist an n-noid symmetric with respect to a horizontal plane, which have a rectangular domain and all ends are on the horizontal plane and all ends lies on a straight line on the domain.