Future Research

- 3d Chern-Simons-matter CFT and dual gravity solution

The programme towards studying gauge/gravity correspondence within the context of AdS4/CFT3 becomes concrete owing to the pioneering work of Aharony, Bergman, Jafferis and Maldacena (2008). They have found how to construct a much more supersymmetric conformal field theory (SCFT) of Chern-Simons-matter (CSM) type. What makes this possible is an elliptic brane setup in Type IIB string theory. After M-theory lift to eleven dimensions, one obtains N M2-branes extending along (012) directions transverse to Cone(M7). This is an eight-dimensional cone filling (345678910) directions.

After N M2-branes backreact such that the flat spacetime gets curved, one finds that the 11d gravity dual is AdS4 ×M7. In [8] of Publication List, we found a new class of examples in AdS4/CFT3. That is, we constructed new 3d N = 3 SCFT on M2-branes whose dual gravity solution becomes AdS4 ×M7. Here, M7 denotes the base tri-Sasakian 7-manifold being Eschenburg space.

This is the first time that Eschenburg space finds its own application within the context of AdS/CFT. We can perform various non-trivial tests of 3d AdS/CFT through known properties of Eschenburg space thanks to mathematicians.

- Uniformization approach to derivation of AGT conjecture

Alday-Gaiotto-Tachikawa (2009) claims that there exists certain duality between 2d Liouville CFT and Nekrasov0s instanton partition function of 4d N = 2 SU(2) Seiberg-Witten theory. Because both two subjects have been developed independently during the past decades, their coincidence is still quite mysterious and amazing.

Inspired by their work, in [4] of Publication List we tried to answer why there should be such a correspondence. Our main approach was by means of Koebe-Klein-Poincar´e uniformization theorem for Riemann surfaces and Fuchsian-type differential equation showing up there. We succeeded in obtaining SU(2) conformal Seiberg-Witten prepotential from these Fuchsian equations.

This might be considered as a derivation of AGT conjecture at least at the level of classical Liouville CFT. What we were after are two elements: Polyakov conjecture and arguments in Hermitian matrix model. In [2,3,5,6] we also explored various new relations between Seiberg-Witten theory and integrable model. We believe that this sort of 4d/2d duality will launch another concept revolution in both mathematics and physics. All these show the depth of string/M-theory.