

The plan of our study

Every link can be realized as a closed braid. A. Kawauchi suggested the following project: enumerate the prime links by using the braid expressions and enumerate the closed connected orientable 3-manifolds by using the prime link table. We have enumerated the 3-manifolds with lengths up to 10, according to this project.

We describe the outline of the project. A well-order (called a *canonical order*) was introduced on the set of links by A. Kawauchi [K] (see also A. Kawauchi and I. Tayama [KT1]).

We assign to every link a lattice point whose length is equal to the minimal crossing number on closed braid forms of the link and we call the number the *length* of the link. We note that a link L is smaller than a link L' in the canonical order if the length of L is smaller than that of L' , and for any natural number n there are only finitely many links with lengths up to n . Let \mathbf{L}^p be the set of prime links and \mathbf{M} the set of closed connected orientable 3-manifolds. Let $\chi : \mathbf{L}^p \rightarrow \mathbf{M}$ be a map defined by $\chi(L) = \chi(L, 0)$ (that means the result of the 0-surgery on S^3 along L). Then it is known that χ is surjective and A. Kawauchi defined a map $\alpha : \mathbf{M} \rightarrow \mathbf{L}^p$ by $\alpha(M) = \min\{L \in \chi^{-1}(M) : L' \in \chi^{-1}(M), \pi_1(E(L)) = \pi_1(E(L')) \Rightarrow L \leq L'\}$ for $M \in \mathbf{M}$, where $E(L)$ is the exterior of L . By using α , we consider \mathbf{M} as a subset of \mathbf{L}^p and introduce the well-order into \mathbf{M} .

As we said, we have completed the table of 3-manifolds with lengths up to 10. There are two plans for our study:

- (1) we enumerate the 3-manifolds with lengths up to 11,
- (2) we enumerate special manifolds with lengths up to a large number.

According to the second plan, we enumerate the homology spheres until Poincare sphere appears.

References

- [K] A. Kawauchi, A tabulation of 3-manifolds via Dehn surgery, *Boletin de la Sociedad Matematica Mexicana* (3) 10 (2004), 279–304.
- [KT1] A. Kawauchi and I. Tayama, Enumerating prime links by a canonical order, *Journal of Knot Theory and Its Ramifications* Vol. 15, No. 2 (2006) 217–237