Research summaries

The Keller-Segel model on chemotactic aggregation of slime mold is described as a drift-diffusion system. For this system it is well-known that the finite-time blowup of nonnegative solutions can occur according to space dimensions. In particular it is suggested that the total mass of solution plays an important role in the structure of nonnegative solutions in two dimensional case.

I have investigated the asymptotic behavior of solutions to the following Keller-Segel system in the whole space:

$$
\text{(KS)} \qquad \begin{cases} \partial_t u = \Delta u - \nabla \cdot (u \nabla v), & x \in \mathbb{R}^n, t > 0, \\ \partial_t v = \Delta v - v + u, & x \in \mathbb{R}^n, t > 0, \\ u(x,0) = u_0(x), v(x,0) = v_0(x), & x \in \mathbb{R}^n. \end{cases}
$$

In my research the following results were obtained:

I. It is well-known that every bounded solution to (KS) decays to zero and behaves like the heat kernel as $t \to \infty$. Professor Toshitaka Nagai and I improved the above results and gave the asymptotic profiles of such solutions up to the second order in terms of the heat kernel ([1, 8]).

II. In the case $n \geq 2$, I gave the higher-order asymptotic expansions of solutions to (KS) with special space-time decay properties by the method used in Navier-Stokes equations. Also it is observed that there appears a difference between the expansions in even and odd dimensional cases ([2]). This phenomenon do not occur for Navier-Stokes equations.

III. For bounded solutions to (KS), if the l-th moment of u_0 is finite for some $l \geq 1$, then $u(t)$ admits moment estimates of l-th order. Moreover, as an application, I find a reasonable method to deduce the higher-order asymptotic expansions of such solutions ([3, 7]). Therefore it is seen that the special space-time decay properties on the solutions to (KS) in [2] need not be imposed in order to obtain these higher-order asymptotic expansions. Also the method in [3, 7] is applicable for other nonlinear drift-diffusion equations.

IV. I proved that the rates of convergence to the heat kernel of decaying solutions to (KS) obtained in the previous works can be improved by taking into account the center of mass of such solutions. Also I discussed the optimality of convergence rates in one or two dimensional cases ([4], [5]). It is observed from the above results that there is a deep relation between the center of mass and asymptotic behavior of solution to (KS). Hence it is suggest that there appears such a relation in other nonlinear partial differential equations.

V. We consider the following Keller–Segel system:

$$
\text{(DD)} \qquad \begin{cases} \partial_t u = \Delta u - \nabla \cdot (u \nabla v), & x \in \mathbb{R}^2, t > 0, \\ 0 = \Delta v + u, & x \in \mathbb{R}^2, t > 0, \\ u(x,0) = u_0(x) \geq 0, & x \in \mathbb{R}^2. \end{cases}
$$

Recently, Professor J. López-Gómez, Professor Toshitaka Nagai and I proved that there exists the unique nonnegative solution to (KS) if a nonnegative function u_0 is in L^1 with $\int_{\mathbb{R}^2} u_0 \, dx = 8\pi$, and the finiteness of the relative entropy of the fast diffusion equation is assured. Moreover they showed that such solutions converge to the steady-states for (KS) ([6]).