

Research Plan

Atsushi Yamamori

In my previous research, I considered the Hartogs domain under the following condition:

Condition. Let p be a positive continuous function on Ω and $s > 0$. The weighted Bergman kernel K_{Ω, p^s} is expressed as

$$K_{\Omega, p^s}(z, z') = \chi(s)F(z, z')^{-s}G(z, z') \quad \text{for all } (z, z') \in \Omega \times \Omega,$$

where χ is a polynomial in s , F, G are functions on $\Omega \times \Omega$ and the functions F, p satisfy $p(z) = F(z, z)$ and $|F(z, z')|^2 \geq F(z, z)F(z', z')$ for all $z, z' \in \Omega$.

We already know that if Ω is \mathbb{C}^n or a irreducible bounded symmetric domain then the pair (Ω, p) satisfies Condition for a certain p . However I do not know any other examples.

- It would be interesting to find other concrete examples which satisfy Condition.

In my previous work, I used the Forelli-Rudin construction(Ligoeka, 1989) to obtain an explicit formula of the Hartogs domain. It is known that there are some generalizations of the Forelli-Rudin construction (M. Englis, G. Zhang). And recently, I found another direction of generalization.

- It would be interesting to generalize my previous works by using these generalizations.

There are many works for the Cartan-Hartogs domains:

1. The comparison theorem for the Bergman and the Kobayashi metrics on the Cartan-Hartogs domain (by X.Zhao, D.Li, W.Yin).
2. The Kähler-Einstein metric on the Cartan-Hartogs domain (A.Wang, M.Wang, L.Zhang).
3. The Balanced metric on the Cartan-Hartogs domain (A.Loi, M.Zedda).

- It would be interesting to generalize these works for the Hartogs domain which satisfies Condition.