Research plan

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(A). An algebra is said to be *tame* if all but only finitely many isoclasses of indecomposable modules with the same dimension are controlled by only finitely many parameters. Note that any representation-finite algebra is tame by the 2nd Brauer-Thrall conjecture. And it is known that the representation type of algebras can be divided into tame and wild (Drozd). For a tame selfinjective algebra, we have the following conjecture in connection with the representation dimension.

Conjecture. *Any tame selfinjective algebra has stable dimension at most* 1.

Under a certain assumption, an upper bound of the stable dimension of a standard selfinjective algebra is given by the derived dimension of algebras having finite global dimension [3]. By using this fact, I verified that any standard selfinjective algebra of polynomial growth has stable dimension at most 1, and hence I obtained an evidence for the conjecture above. Moreover, applying this fact to a selfinjective algebra of wild canonical type, it turns out that an upper bound of its stable dimension is given by the derived dimension of a wild canonical algebra. Any canonical algebra has global dimension at most 2. Thus, I will first consider the question whether the derived dimension of a wild canonical algebra is 2. And then I will decide the stable dimension of a selfinjective algebra of wild canonical type. In consideration of the conjecture above, I hope that the selfinjective algebra has stable dimension 2.

(B). According to Rouquier, an exterior algebra $\wedge(k^n)$ has representation dimension $n+1$, derived dimension n and stable dimension $n-1$. On the other hand, any representation-finite selfinjective algebra has representation dimension 2, derived dimension 1 and stable dimension 0 (Auslander, Chen-Ye-Zhang and Han). Therefore, we have two natural questions.

Question 1. Is the difference between the representation dimension and the derived dimension at least 1 ?

Question 2. Is the difference between the derived dimension and the stable dimension at least 1 ?

At present, it is not known whether there exists an selfinjective algebra such that these dimensions coincide. Oppermann also pointed out them. Thus, the second purpose is to investigate these questions, and then to understand the representation theoretic properties of selfinjective algebras having low derived dimension.

At the beginning, I will consider the natural question whether a selfinjective algebra having derived dimension 1 is representation-finite. After this, I will decide the derived dimensions of selfinjective algebras, whose stable dimensions have already decided.

(C). For an Iwanaga-Gorenstein algebra, the category of Cohen-Macaulay (=CM) modules is a Frobenius category, and then its stable category forms a triangulated category. Hence, the category of CM modules must be important in representation theory. Note that the dimension of its stable category is equal to the stable dimension. If an Iwanaga-Gorenstein algebra is of finite CM representation type, then it has stable dimension 0. Then a natural question arises as to whether the converse should also hold. Thus, the third purpose is to investigate this question. This is a generalization of my result on selfinjective algebras having stable dimension 0.