Research statement

Cohomologies of weighted Grassmannians

As joint works with Tomoo Matsumura (KAIST), we computed the structure constants for the rational cohohomology of the weighted Grassmannian with respect to Schubert classes, and a presentation of the rational cohomology ring as a quotient of a polynomial ring whose variables are Chern classes of the tautological orbi-bundle will be appeared in the revised version of [3]. As next problems, I am interested in the integral cohomology ring and the Chen-Ruan cohomology ring.

For the integral cohomology, it should be considered first whether there are torsion elements in the integral cohomology of weighted Grassmannians. After that, I will ask for a set of generators of the ring and their relations. It may be important to see whether the *divisive* cases are easier to study as in the paper written by Bahri-Franz-Ray for the weighted projective spaces. Since the weighted Grassmannian is an orbifold, giving a presentation of the Chen-Ruan cohomology ring and its relation with Schubert calculus are also an interesting problem. The ultimate goal of this project would be the understanding of the quantum orbifold cohomology of weighted Grassmannians and its relation with Schubert calculus.

In the revised version of [3], we will introduce improved weighted analogue of the Schur polynomials which can be written as linear sums of Schur polynomials with non-negative integer coefficients. It would be interesting to find a way of realizing these representations of the general linear group in a geometric context.

Toric manifolds associated to root systems

Let G be a semisimple algebraic group and B its Borel subgroup. Studying the subvarieties of the flag variety G/B, called *Hessenberg varieties*, is recently an active research area. These subvarieties are studied from topology, geometry and representation theory. Among these, there is a distinct subvariety, the toric manifold associated to the Weyl chambers of the root system of G. The intersections with Schubert cells of the flag variety provide a cell decomposition of the toric manifold, and hence give an additive basis of the cohomology. Since the intersection theory on the toric manifold computes these structure constants, it is natural to ask for a combinatorial formula of the structure constants with respect to these basis in terms of the root system.

Since the fan consisted of the Weyl chamber is a normal fan of the moment polytope of the flag variety G/B which is Delzant, we can also consider the moment-angle manifold of this polytope. It is also natural to ask whether we have a simple combinatorial expression for Euler numbers, Betti numbers, bigraded Betti numbers of our moment-angle manifold in terms of the root system. Also, since the Weyl group acts on our moment-angle manifold as above, it would be interesting to study the representation induced on their cohomology, as Stembridge studied the Weyl group representation on the cohomology of the above toric manifold.