Research statement (project)

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Over the next few years I am planning to work about the following projects.

To unify toric manifolds and toric HK manifolds from topological point of view. Toric HK manifolds themselves are quite different objects from toric manifolds. However, similar phenomena often occurs between them: for example, their equivariant cohomology rings are isomorphic to the Stanley-Reisner rings of their corresponding combinatorial objects; or equivariant cohomological rigidity holds for both of them. Under the motivation of "to unify toric and toric HK manifolds from topological point of view", I am studying some class which may be regarded as topological generalization of toric HK manifolds in (32), Using the result in (17), we can get the equivariant cohomology rings of them. This study also may be regarded as the geometric counterpart of the study of hypertorus graphs. Over the next few years, I would like to study them from both of geometrical and topological point of view more deeply, and I aim to find the quaternionic analogue of torus manifolds.

Extended actions of GKM manifolds. According to Wiemeler and (14), in general, if torus manifolds have extended actions, then their transformation groups are SU or SO-types. So we are naturally led to ask the problem "what kind of classes of manifolds have the extended actions with other types?" GKM manifolds could be one of such classes. Because all of the homogeneous spaces G/H with same ranks are GKM manifolds, all root systems of type A_ℓ–G_2 can be appeared as the extended actions on GKM manifolds.

In my progress work, I study extended actions of GKM manifolds with several mathematicians via the combinatorial structure of GKM graphs. For example, I think I can define "Cox ring of GKM graphs" by translating the definition of Cox rings of toric manifolds (or other varieties). Next project is to define root systems on this ring. In fact, it seems to be hard to define root systems on equivariant cohomology for the other GKM graphs, unlike the case of torus graphs. So, we need to use more general objects which contain more information than equivariant cohomology.

Moreover, unlike torus manifolds, we can consider the extension of torus actions to the higher dimensional torus actions in GKM manifolds. I am working with Park about the obstruction of this extension.

Classification problems for several classes. I will study the next topic related to rigidity and classification problems.

(1) Cohomological rigidity problem of (quasi)toric manifolds;
(2) Characterization of torus manifolds which satisfy cohomological rigidity;
(3) Characterization of torus manifolds induced from torus graphs (This is working on progress with Karshon);
(4) Homotopical rigidity problem of $\mathbb{C}P$-towers;
(5) Characterize when $\mathbb{C}P$-tower satisfies cohomological rigidity or rigidity by other cohomology theories (I and Ray are working on progress for KO theory);
(6) Cohomological rigidity problem of toric HK manifolds with fixed dimension;
(7) Cohomological rigidity problem of aspherical small covers.

In particular, the third problem is related to the open problem "characterization of multi-fans induced from torus manifolds". So, I believe the third problem is one of the most important problem in toric topology.

In this year, I am planning to stay in Toronto until September and work with Professor Yael Karshon. Moreover, I am planning to work with Naohiro Kato staying in UK (he is also a OCAMI researcher) about geometric structures of small covers.