Research plans

MUROYA, Hisayoshi

Research in Euclidean three space

It is an interesting problem to observe the moduli space of n-noids of genus one. However, this problem is difficult because of the period condition with respect to the homology basis of the torus. Therefore we consider n-end catenoids, which have a symmetry with respect to a plane and a certain arrangement of ends. By the symmetry, the domains are restricted to rectangular or rhombic tori. In this condition, we present two problems:

1. Construction of examples which are important to observe the moduli space of n-noids

Here we consider n-end catenoids with n-gon prism symmetry. By the symmetry, the angles of neighboring ends must satisfy $360° \times k/n$, where k and n are relatively prime positive integers and $k \leq n-1$. In a recent work, I prove that "there exists an n-end catenoid defined on a rectangular torus with n-gon prism symmetry if and only if the angle of neighboring ends is less than 180°." We can consider this result as a kind of generalization of R. Schoen's work, that is, the nonexistence of "the catenoid of genus one". I also prove that such an n-end catenoid is unique for each k and n. On the other hand, a result of numerical analysis shows that the condition "the angle of neighboring ends is less than 180°" is not sufficient but necessary for the existence of an n-end catenoid defined on a rhombic torus. It is a goal to see this result in more detail.

2. Boundary of the moduli space of n-noids

It is well known that a subsequence of arbitrary n-noids converges to a union of (branched) minimal surfaces with finite total curvature. Henceforth we use the word "collapse" in the sense that a limit of a sequence of n-noids is separated into two surfaces at least.

By R. Schoen's work above, if the flux data of an n-end catenoid of genus one goes near to that of "the catenoid of genus one", then the surface must collapse. We constructed new examples of families, whose flux data go near to that of "the catenoid of genus one", they are, a family of 4-end catenoid with cuboid symmetry and two families of 3-end catenoids with isosceles triangular prism symmetry. First of all, we would like to estimate the flux of the handles of such surfaces, and observe the concrete collapse. Thereafter we would also like to observe the collapse that is caused by the collapse of the torus domain, for example, the collapse of the family of $2N$-end catenoid with regular N-gon prism symmetry.

Research in Minkowski three space

T. Imaizumi and S. Kato defined n-noids in Minkowski three space as maxfaces with only simple ends of order two. They also give a formulation of n-noids of genus zero, and by using the formulation, they solve an inverse problem of flux in the case that the number of ends is at most four.

3. Formulation and examples of n-noids of genus one in Minkowski three space

We would like to formulate n-noids of genus one in Minkowski space and research similarities and differences between Euclidean and Minkowskian case. To achieve our goal, we follow the formulations of n-noids of genus zero, which are given by S. Kato, M. Umehara and K. Yamada in Euclidean case, and by T. Imaizumi and S .Kato in Minkowskian case.