Research results

MUROYA, Hisayoshi

I have studied an n-noid, that is, a complete minimal surface in Euclidean three space with finite total curvature, and all of whose ends are embedded. Here n is a total number of ends.

Backgrounds

By the Weierstrass representation formula, n-noid is given by the Weierstrass data, which is a pair of a meromorphic function and a meromorphic 1-form on a compact Riemann surface. Conversely, for the Weierstrass data on a Riemann surface, an n-noid given by this data is well-defined if and only if the period condition is satisfied.

We can define weights for embedded ends. If the weight of an embedded end is zero (resp. is not zero), this end is called a planar end (resp. a catenoidal end). In particular, an n-noid is called an n-end catenoid, if all the ends are catenoidal.

Let X be an n-noid, $q_1, \ldots, q_n$ the ends of X, w_j the weight of q_j, and G the extended Gauss map of X. Then the pair $(w_1, \ldots, w_n) \in \mathbf{R}^n$ and $(G(q_1), \ldots, G(q_n)) \in (\mathbf{S}^2)^n$ is called the flux data of X. By the Gauss' divergence formula, we have $\sum_{j=1}^{n} w_j G(q_j) = 0$. Conversely, we can consider a problem of finding an n-noid that realizes given data w_j and $G(q_j)$ satisfying $\sum_{j=1}^{n} w_j G(q_j) = 0$. Such a problem is called an inverse problem of the flux. In the case of genus zero, S. Kato (OCU), M. Umehara (TITECH) and K. Yamada (TITECH) expressed the period condition as a system of algebraic equations with respect to the flux data, and proved that for almost all flux data, there exists an n-end catenoid realizing the given data.

Summary of results

Here I describe the results of a joint work with S. Kato on n-noids of genus one. We observe zeros and poles of elliptic functions, and conclude that n-noids are classified to two classes $\omega = \omega_2$ and $\omega = 0$.

(The class $\omega = \omega_2$) The well-known examples with catenoidal ends, such as Costa's example and Berglund-Rossman's example, are in this class. We give a formulation of n-noids in this class by using functions on annular domains, and express the period condition as a system of equations with respect to the flux data (cf. [1]). By using this formulation, we construct a family of $2N$-end catenoids ($N \geq 2$) with regular N-gon prism symmetry (cuboid symmetry for $N = 2$), and two families of 3-end catenoids with isosceles triangular prism symmetry. Here, all of these examples are defined on rectangular tori.

(The class $\omega = 0$) In this class, the known examples are Costa's 4-noid and its generalization by Kusner-Schmitt. However, there are no previously known examples with catenoidal ends in this class. We give a formulation of n-noids in this class by using the Weierstrass zeta function on parallelograms, and express the period condition as a system of equations with respect to the flux data (cf. [2]). Observing covering spaces enables us to treat these two classes equally. By using this method, we construct $2N$-end catenoids with regular N-gon prism symmetry ($N \geq 3$, N: odd), whose domains are rhombic tori. In particular, this family contains two N-end catenoids for $N \geq 5$. On the other hand, we also prove that there does not exist an n-noid defined on a rectangular torus with plane symmetry and certain arrangement of ends in this class.