Summary of My Research Interests

Yuko Nagase

Current Research

My research is about mathematical analysis of interfaces which appear between two phases in several physical phenomena, such as crystalline surface. There are several directions to study such a kind of problem, I'm especially focusing on two characteristic points, one is "anisotropy" of curvature of interface. The property "anisotropy" is very important to comprehend natural phenomena, although its treatment becomes quite complicated and difficult. We need to approach not only from theory of partial differential equations but also by understanding its geometrical construction. The second point is its framework. We consider such an analysis within a framework of "geometric measure theory."

I had studied general theory of partial differential equations and some basics of geometric measure theory during Ph.D course and I have studied several problems concerning to the Ginzburg-Landau type energy functionals for real valued functions, which is called Modica-Mortola type energy. Moreover, since I started to be a post doctoral fellow, I have studied about mathematical analysis of an energy functional adding the effect of anisotropy and also several problems adding the influence of a white noise. I will state more details for each topic in the following;

In the first paper "**Interior gradient estimate for 1-D anisotropic curvature flow,**" we consider *anisotropic curvature flow*, which is a surface evolution equation and a generalization of mean curvature flow. We establish the interior gradient estimate in one dimensional case. The estimate depends only on the height of the graph and not on the gradient at initial time. The proof relies on the monotonicity property of the number of zeros for the parabolic equation by S.B. Angenent.

In the second paper "**A singular perturbation problem with integral curvature bound,**" We consider a singular perturbation problem of Modica-Mortola energy functional as the thickness of diffused interface approaches to zero. For a variational problem of this energy functional under volume constraint, it is well-known that this energy functional Γ-converges to a perimeter functional of the limit interface by Modica-Mortola or Sternberg. Moreover, the Lagrange multiplier converges to a constant mean curvature of the limit interface by Luckhaus and Modica. We are interested in not only for the minimizers but also for general critical points. We assume that sequence of functions have uniform energy and square-integral curvature bounds in two dimension. We show that the limit measure concentrates on one rectifiable set and has square integrable curvature. This problem is a modification of one of the conjectures by De Giorgi and the result is very useful and applicable to analysis of interfaces, for instance, a switching problem of a stochastic Allen-Cahn equation in my 4th paper and study of Brakke's motion as a singular limit of the Allen-Cahn equation.

The third paper **" The Gibbs-Thomson relation for non homogeneous anisotropic phase transitions, "** this work is done during a post-doc. in the University of Naples. In this paper, we prove the Gibbs-Thomson relation between the coarse grained chemical potential and the non homogeneous and anisotropic mean curvature of a phase interface within the gradient theory of phase transitions thus proving a generalization of a conjecture stated by Gurtin and proved by Luckhaus and Modica in the homogeneous and isotropic case. We consider a minimization problem for non homogeneous anisotropic version of the Modica-Mortola type energy under volume constraint, and as result, we proved the Lagrange multiplier converges to a constant anisotropic curvature of the limit interface. The anisotropic curvature is formulated by using a framework of the Finsler metrics.

In the paper **" Action minimization for an Allen-Cahn equation with an unequal double-well potential,"** We consider the minimization problem of the Allen-Cahn action functional with an " *unequal"* double-well potential. For a deterministic Allen-Cahn equation, there are two stable states. If we consider the Allen-Cahn equation with a white noise, switching from one deterministic stable state to the other rarely occurs. It is well-known that probability of switching is determined by the minimum of the action (or cost) functional by the large deviation theory. We give an explicit description of the minimum and its optimal path in one-dimension. For this analysis, the treatment of interfaces are very important and we apply the result in my second paper.

In the latest paper **" On the existence of solution for a Cahn-Hilliard / Allen-Cahn equation,"** we consider a Cahn-Hilliard/Allen-Cahn equation, which is introduced in Karali and Katsoulakis as a simplification of a mesoscopic model for multiple microscopic mechanism in surface process. This equation is a gradient flow in a suitable metric for the Modica-Mortola type energy. In a suitable scaling, its singular limit is mean curvature flow, similarly to Allen-Cahn equation, but with a different mobility. We give an existence of the solution, slightly improved from the result by Karali and Ricciardi. We also present a stochastic version of this equation, which will be partially including a forthcoming paper.