Results

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The problem to construct a distinguished metrics on a manifolds is important in differential geometry. The Einstein structures and the Ricci soliton structures are candidates to this question. I study the Ricci soliton structures.

If g_0 satisfies

$$\mathrm{Ric}[g_0] = cg_0 + L_X g_0 \ ,$$

where Ric is the Ricci tensor of g_0, X is a vector field and c is a constant, then (M^n, g_0, X, α) is called a *Ricci soliton structure* and g_0 *the Ricci soliton.* A Ricci soliton is a Ricci flow solution $\frac{\partial}{\partial t} g(t)_{ij} = -2Rc[g(t)]_{ij}$.

In general, problems for Ricci solitons are second-order differential equations. Lauret introduced algebraic Ricci solitons in Riemannian case.

Definition 1. *Let (G, g) be a simply connected Lie group equipped with the left-invariant pseudo-Riemannian metric g, and let $\mathfrak{g}$ denote the Lie algebra of G. Then g is called an algebraic Ricci soliton if it satisfies*

$$\mathrm{Ric} = c\mathrm{Id} + D \tag{1}$$

where Ric *denotes the Ricci operator, c is a real number, and* $\mathrm{D} \in \mathrm{Der}(\mathfrak{g})$ *(*D *is a derivation of $\mathfrak{g}$), that is:*$\mathrm{D}[X, Y] = [\mathrm{D}X, Y] + [X, \mathrm{D}Y]$ *for any* $X, Y \in \mathfrak{g}$. *In particular, an algebraic Ricci soliton on a solvable Lie group, (a nilpotent Lie group) is called a solvsoliton (a nilsoliton).*

Lauret proved that sol-solitons on homogeneous Riemannian manifolds are Ricci solitons. Algebraic Ricci solitons allow us to construct Ricci solitons in an algebraic way, i.e., using algebraic soliton theory, the study of Ricci solitons on homogeneous manifolds becomes algebraic.

I studied algebraic Ricci solitons on solvable Lie groups in the Lorentzian case ([3]). I constructed Lorentzian solvsolitons on H_3, $E(2)$, $E(1,1)$, H_N, and $G_m(\lambda)$. I obtain the shrinking solvsolitons on H_3, $E(2)$, $E(1,1)$, H_N, and the steady solvsoliton on $G_m(\lambda)$ without Riemannian analog. In [4], Wafaa Batat and I studied algebraic Ricci solitons in three-dimensional Lie groups and gave the complete classification. In particular, we obtain a new solitons and we prove that there exist Ricci solitons but not algebraic Ricci solitons on $SL(2, \mathbb{R})$. In [5], Wafaa Batat and I classified the algebraic Ricci solitons on four-dimensional pseudo-Riemannian generalized symmetric spaces. In [6], Phillip E. Parker and I studied H-type in the Lorentzian setting and constructed nilsolitons. In [7], Wafaa Batat and I studied second-order symmetric 4-dimensional spacetimes and constructed Ricci solitons and Yamabe solitons.