The previous results

0. Motivations. The applicant has been studied the scattering theory for quantum many problems. We study the asymptotic behavior of the dynamics of quantum many body systems as time goes to infinity. Such systems are described by Schrödinger equation. In mathematical point of view, we study the asymptotic behavior of the solution of Schrödinger equation. The solution can be written by using the time evolution group generated by Hamiltonians (in the following we denote it by H), which are self-adjoint operators on some Hilbert space. Hence we need to study the spectral property of such Hamiltonians. In the present setting, the Hilbert space we use is generally infinite dimensional. Therefore, a careful study must be required. So far, the mathematical scattering theory for the system of *finite* quantum particles has been studied by many researchers. On the other hand, quantum phenomena for the system of infinite particles, such as BEC (we will mention later on), has been studied by many theoretical physicists recently. To treat the scattering theory for infinite systems, the framework of quantum field theory must be required. To the author's knowledge, there are very few results about this topic.

1. Results. So the author studied the scattering theory for quantum field theory. In view of physics, one of the most important objects is scattering matrix. It relates relates the initial state and the final state of a physical system undergoing a scattering process. Mathematically, the existence of scattering matrix is derived from proving asymptotic completeness for H. However, it is in general difficult to prove it and the details of spectral properties of H are required. So we study the spectral properties of Hamiltonians for infinitely many boson systems. We have the following three results:

1-a. Limiting absorption principle for the second quantization of self-adjoint operators. In quantum field theory, the physical Hilbert space is called Fock space, which is an infinite direct sum of Hilbert spaces. The free energy is self-adjoint operators of Fock space, which is the second quantization $d\Gamma(T)$of free energy T of one particle. As is well known, the time evolution operator of the system can be represented as the Laplace transform of the operator, which is called resolvent. It is operator valued analytic function on upper and lower half plane of complex numbers. If it can be continued meromorphicaly from upper half plane to lower half plane along real axis, then we can get the decay estimates of time evolution operators as time goes to infinity. This fact is said to be the limiting absorption principle (LAP in short) for H. We proved that, under rather general settings, $d\Gamma(T)$ satisfies LAP.

1-b. Essential spectrum for H. Next we considered the spectral analysis of infinitely many boson systems with pair interactions. The Hamiltonians of such systems are defined by the sum of second quantization of self-adjoint operator (unperturbed Hamiltonian) which we study above and potential (perturbation). This operator can be regarded as a generalization of many body Schrödinger operators. However, the perturbation is not relatively bounded to unperturbed Hamiltonian. Hence we can not apply the perturbation theory of T.Kato. To overcome the difficulty, we proved so called Higher order estimates, to prove that the location of essential spectrum for H

1-c. A trace formula via functional integration method. Partition function plays an important role in quantum statistical physics, which can be formulated by the trace of heat semi-group generated by H. We derived the trace formula by path integral method and considered the semi-classical limit of it.