1 英訳

In the doctor course, I was studying the higher-dimensional Weil-zeta function, which is defined as the generating function of the number of effective cycles of intermediate dimension on a fixed polarized projective variety. I showed that the convergent radius of the higher-dimensional Weil-zeta function is determined by the self-intersection number of the polarization divisor, when the dimension of effective cycles are of codimension 1 relative to the given variety [1].

Also, some computations hint us that when the codimension of the cycles is more than 1, then this convergent radius gives a piecewise-linear function on the ample cone of the Néron-Severi group, which implies some relations with tropical geometry.

With these aspects in mind, I aimed to construct the theory of $\mathscr{A}$-schemes which includes tropical geometry and schemes over $\mathbb{F}_1$ in 2010. I have rewritten the paper, so as to make the theory more accessible [2]. This $\mathscr{A}$-scheme is defined as an $\mathscr{A}$-valued space with nice properties, for a fixed algebraic type $\mathscr{A}$ which satisfies certain conditions (which is fulfilled, for example, when $\mathscr{A}$ is the type of monoids, semirings, and rings). When we restrict our attention to the case when $\mathscr{A}$ is the type of rings, then the $\mathscr{A}$-schemes share many properties with ordinary schemes, but on the other hand, have a remarkable advantage that we can take infinite limits of them [3].

For example, we can construct Zariski-Riemann spaces (ZR-spaces for short) as $\mathscr{A}$-schemes, by just imitating the construction of Stone-Čech compactifications. We can obtatin the Nagata embedding theorem as a corollary, using ZR-spaces. Also, any separated scheme (possibly not of finite type) can be embedded in a ZR-space in the category of $\mathscr{A}$-schemes.

Pushing forward this geometric construction further, we succeeded in constructing pure-algebraically the arithmetic compactification $\overline{\operatorname{Spec}\mathbb{Z}} = \operatorname{Spec}\mathbb{Z} \cup \{\infty\}$ of the spectrum of rational integers [4]. There are preceding works done by Haran and others on this topic; however, our construction has the advantage that the construction can be characterized by the universal property, which naturally generates the archimedean place from the algebraic structure. This supports various concepts of Arakelov geometry, and we are hopeful that this can be applied to various theories in arithmetics.