Research program

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The harmonic Bergman space

$$b^p(\Omega) := \{u : \Omega \to \mathbb{R} \mid \Delta u = 0 \text{ かつ } \|u\|_{L^p} < \infty\}$$

depends on the following four contents: (1) the range of index p, (2) the property of a domain Ω, (3) L^p-norm (measure), (4) the differential operator Δ. In our previous results, we focused two contents (1) and (2). It is interesting to focus (3) or (4).

1. Weighted harmonic Bergman spaces

Weighted harmonic Bergman space is defined by the space of all harmonic functions which are p-th integrable function with respect to the weighted norm. This space has the reproducing kernel and its kernel is called the weighted harmonic Bergman kernel. Recently, M. Engliš studied the boundary behaviors of the weighted harmonic Bergman kernel of smooth bounded domains by using pseudo differential operator calculus. As an application of M. Engliš result, I try to analyze Toeplitz operators by using similar arguments in [2].

2. A space consisted of the solution for the heat equation

M. Nishio, K. Shimomura and N. Suzuki defined the α-parabolic Bergman space on upper half-space, which is the set of all solutions for α-parabolic equation (which includes heat equation) and analyzed Toeplitz oerators on α-parabolic Bergman spaces. I try to define $alpha$-parabolic Bergman spaces for another domains by using the pseudo differential operator calculus.

3. Harmonic Fock spaces

We define the Fock space by the set of all entire functions which are square integrable function with respect to some Gaussian measure. Fock space is a central subject in quantum physic and many researchers have been studied the Fock space. The harmonic Fock space is defined by the set of all harmonic functions on $\mathbb{R}^n$ which are square integrable function with respect to some Gaussian measure. There are few papers for the harmonic Fock space. I try to complete the harmonic Fock space theory.