a Colored braided homology

In the PhD thesis and satellite papers, I constructed a general *braided homology* theory and showed it to include familiar homology theories for various algebraic structures (associative and Lie/Leibniz algebras, (bi)quandles/(bi)racks, etc.). The next step consists in developing a colored version of this theory, with several applications in mind:

- ✓ *Multi-term distributive homology*, recently introduced by J. Przytycki and A. Sikora as a generalization of rack homology, should be naturally interpreted within the colored braided homology. This would give a powerful tool for studying distributive homologies and make their applications to knot theory intrinsic, since knot theory is related to braided homology via braid theory.
- ✓ Colored braided homology should also be applicable to *G-families of quandles*, recently designed by A. Ishii, M. Iwakiri, Y. Jang, and K. Oshiro for constructing powerful handlebody-knot invariants. One would thus get a homology theory for G-families potentially different from the original one, and open to exploration.

I am also working on several generalizations of the Ishii-Iwakiri-Jang-Oshiro work, namely its virtual/twisted versions; its biquandle version; and a theory of families over structures other than groups. As for more topological aspects of *handlebody-knot* theory, I would like to develop a related theory of *"braids with trivalent vertices"* – in particular, to extend Alexander and Markov theorems to them, as it was done for virtual braids by S. Kamada.

b Virtual biquandle cocycle invariants

S. Carter, M. Elhamdadi, and M. Saito generalized Carter-Jelsovsky-Kamada-Langford-Saito *quandle cocycle knot invariants* by working with the more general biquandle structure. J. Ceniceros and S. Nelson extended the resulting *biquandle cocycle invariants* to virtual knots by constructing a Reidemeister-move-inspired cohomology theory for *virtual biquandles* (in the sens of Kauffman-Manturov). They also predicted a deeper cohomology theory explaining why their topology-driven construction actually works. Such a theory is naturally formulated inside my *braided system* framework, since a virtual biquandle can be viewed as a braided system. A *welded biquandle* also admits a braided system interpretation, hence a braided homology theory. The work on these interpretations and their applications is in progress.

c Virtual braid groups

One more research direction I would like to pursue, still in the field of virtual knot theory, concerns the *faithfulness of virtual braid group actions on free virtual self-distributive structures* (in the sense of V.O. Manturov). I have obtained some partial results in favor of the faithfulness in the *free monogenerated virtual shelf* case. Motivation comes from usual braid group theory, where P. Dehornoy's work on the actions on free shelves lead, in a very spectacular way, to an explicit left-invariant *order on the braid groups*, with in addition unexpected connections to the theory of *large cardinals*. The action on *free virtual quandles* is also of interest, since its freeness is equivalent to a *conjecture of Manturov* which is still open.