Research Results

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• 2d-4d connection

The 2d-4d connection states the equivalence between the conformal block in 2-d CFT and the instanton partition function in 4-d supersymmetric gauge theory. This connection is not proved yet. However the contradicted result with this connection have never been obtained and there is the proof in the special cases. The connection between the q-W_n conformal block and the 5-d instanton partition function was also proposed. These have two parameters q and t. By taking the limit $q, t \to 1$, this connection reduces to the 2d-4d one. I have studied the root of unity limit in q and t. The generators of the q-Virasoro (q-W_2) algebra can be described by the q-boson field. The generators of the superconformal algebra appear in the $q, t \to -1$ limit. Similarly, the free boson and free fermion which describe this algebra and construct the block can naturally be obtained from the q-boson in this limit. Moreover, in the r-th root of unity limit, $\mathbf{Z}_r$-parafermions appear and then the fractional superconformal algebra can be constructed. On the 5-d side, I obtained the 5-d instanton partition function in the root of unity limit which is the same as that used on the 2-d side. I have confirmed that the results are equal to the 4-d ALE instanton partition function at the lower level at least. The 2d-4d connection may be unifiedly understood through the limiting procedure in the 2d-5d one.

The 4-d $\mathcal{N} = 2$ gauge theory can be constructed by using NS5-branes and D4-branes which are ended on the former branes. Those branes are obtained by compactifying appropriately M5-branes in M theory. The stack of M5-branes has six-dimensional world volume and its four-dimensional part realizes a supersymmetric gauge theory. On the other hand, the remaining two-dimensional part plays role of Seiberg-Witten curve and it decides the low energy effective theory of the gauge theory. I have studied the correspondence between gauge theory under 90° rotation of the curve. The clear correspondence between WZW model and XYZ spin chain have been constructed by using Ward-Takahashi identity and Baxter TQ equation.

• Matrix Model

Because the superstring theory is defined on ten-dimensional spacetime, so are the matrix models. In conclusion, if we would like to obtain the models which describe realistic world, the compactification of spacetime to four-dimensions are required. In particular, I studied about the USp matrix model compactified by $\mathbb{C}^3/\mathbb{Z}_3$ and then I succeeded in enumerating all possibilities to be able to be consistently defined.

I calculated the partition functions of reduced matrix models for various gauge groups by using the prescription of Moore-Nekrasov-Schatashvili.

On the other hand, in matrix model, Spacetime points are describe ed by the eigenvalues of the bosonic matrices and then the coordinates are treated as dynamical variables. The distribution of the eigenvalues is, therefore, a very interesting research object and the matrix model has possibility to describe the spacetime where we live. By calculating the one-loop effective action for the eigenvalues I studied the effect of orientifolding to matrix model and showed that the spacetime directional asymmetry of attractive force between eigenvalues appears. The spacetime points are eventually attractive to an imaginary four-dimensional surface. Moreover I calculated two-loop corrections and found that in the case of short distance the interaction between eigenvalues is turned to repulsive. In the USp matrix model, the spacetime points are stabilized near above mentioned four-dimensional plane and then the 4-d spacetime are produced.