Research results

My research interests focus on topology and geometry of spaces with symmetry. Especially, manifolds/vareites with torus actions and their relations with combinatorics are of particular interests. Below, I summarize my research results so far.

The toric manifolds associated with Weyl chambers ([1.2] in the List)

Given a root system Φ, the collection of the Weyl chambers forms a fan, and we get a non-singular projective toric variety $X(\Phi)$. The fundamental question for this space is; describe geometry/topology of $X(\Phi)$ in terms of the root system Φ. We provide a combinatorial rule to compute the intersection numbers for invariant divisors of Φ by Young diagrams. Realizing $X(\Phi)$ in the flag variety as an orbit closure of the action of a maximal torus, we obtain a recursive formula for the structure constants with respect to the basis induced by the Schubert cell decomposition, as an application of the computation of intersection numbers.

Schubert calculus for weighted Grassmannians ([1.5] and [1.1] in the List)

This research is a joint work with Tomoo Matsumura (KAIST). Schubert calculus has its origin in counting the number of lines with certain conditions in a complex affine space. In the language of the modern mathematics, it is a problem of computations of the *structure constants* of the cohomology of the complex Grassmannian with respect to the Schubert classes. Topology, geometry, representation theory and combinatorics meets there via these structure constants. In this project, we developed Schubert calculus for the *weighted Grassmannian orbifold*. In particular, we introduced a natural definition of Schubert classes of the cohomology of the weighted Grassmannian, and computed the structure constants with respect to them. To do that, we in fact formulate the same problem in equivariant cohomology of the natural torus action on the weighted Grassmannian, and calculated the equivariant structure constants. The formula of our structure constants are described by the *equivariant* structure constants of the non-weighted Grassmannian.

A generalization of convexity theorem of moment maps ([1.6] in the List)

For a Hamiltonian torus actions on a compact symplectic manifold, Atiyah and Guillemin-Sternberg showed, in 1982, that the image of the moment map is a convex polytope. It is known that the combinatorial information of the moment polytope reflects the equivariant topology of the symplectic manifold. Kirwan generalized their theorem to the case for Hamiltonian actions of compact connected Lie groups. I gave a different generalization of the original convexity theorem with three Hamiltonian torus actions. The idea of three torus actions was motivated by the research of integrable structures of the Toda lattice by Agrotis-Damianou-Sophocleous. A generalization of the Delzant's classification of the symplectic toric manifolds in terms of moment polytopes is a future problem.