Previous research

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1. Sigma functions of algebraic curves

Weierstras's elliptic sigma functions are generalized to the case of hyperelliptic curves by F. Klein. Klein's sigma functions are defined by modifying the Riemann's theta functions in such a way. One of the prominent features of Klein's sigma functions is their algebraic nature directly related to the defining equation of the algebraic curve. From this, it is known that the sigma functions have some important applications in integrable system and cryptography. For example, the solution of Jacobi's inversion problem for hyperelliptic curves has a simple description by hyperelliptic $\wp$-functions, the second logarithmic derivatives of the sigma functions. We can give the explicit solutions of the KdV equations and sine-Gordon equations in terms of $\wp$-functions. Moreover, in cryptography, a computation method of a pairing with the sigma functions is proposed. Recently, Klein's sigma functions are extended to the case of the plane algebraic curves defined by $y^n = x^s + \sum_{in+js<ns} \lambda_{ij} x^i y^j$ for relatively prime positive integers n and s, which are called (n,s) curves [4]. Nakayashiki [6] gave a formula which expresses the sigma functions of (n,s) curves in terms of the prime forms and showed that the sigma functions are determined algebraically from the defining equation of the (n,s) curves. The applicant extended the sigma functions of (n,s) curves to the case of telescopic curves proposed by Miura [5] and showed that they are also determined algebraically from the defining equations of the telescopic curves [1]. Telescopic curves can be hyperelliptic curves and (n,s) curves as special cases. Moreover, the applicant extended the addition formulae for sigma functions of (n,s) curves to those of telescopic curves, which is joint work with Nakayashiki [2].

2. Theoretical analysis of nonparametric regression

Let (X,Y) be a $\mathbb{R}^d \times \mathbb{R}$-valued random vector. In regression analysis, one wants to predict the value of Y after having observed the value of X, i.e., to find a measurable function f such that the mean squared error $\mathbf{E}_{XY}\left(f(X)-Y\right)^2$ is minimized, where $\mathbf{E}_{XY}$ denotes the expectation with respect to (X,Y). Let $m(x) := \mathbf{E}\{Y|X=x\}$ (regression function), which is the conditional expectation of Y given $X=x$. Then, $m(x)$ is the solution of the minimization problem. In statistics, only the data is available, (the distribution of (X,Y) and m are not available), and one needs to estimate the function m from the data $\{(X_i,Y_i)\}_{i=1}^n$, which are independently distributed according to the distribution of (X,Y). We wish to construct an estimator m_n of m such that the expected L_2 error $R(m_n) := \mathbf{E}_{X^nY^n}\mathbf{E}_X\left(m_n(X)-m(X)\right)^2$ is as small as possible, where $\mathbf{E}_{X^nY^n}$ denotes the expectation with respect to the data. The applicant considered the rate of convergence of $R(m_n)$ for the k-nearest neighbor estimators in case that X is uniformly distributed on $[0,1]^d$, $\mathbf{Var}(Y|X=x)$ is bounded, and m is (p,C)-smooth. It is known that there exists a constant $C>0$ such that $R(m_n) \geq Cn^{-2p/(2p+d)}$ for any estimator m_n. It is an open problem whether the optimal rate $n^{-2p/(2p+d)}$ can be achieved by a k-nearest neighbor estimator for $1<p\leq 1.5$ and $p>1.5$, $d\geq 2$. The applicant showed that the optimal rate can be achieved by a k-nearest neighbor estimator for $1<p\leq 1.5$ and can not be achieved for $p>1.5$ [3].

References

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