Abstract of results Makiko MASE

Consider several mirror dualities, for one of which we state its definition:

Definition (*c.f.* Ebeling-Ploog) Let $B = (0, (f = 0))$ and $B' = (0, (f' = 0))$ be germs of bimodular singularities in $\mathbb{C}^3$. A pair (B, B') of singularities are called **transpose dual** if the following three conditions are satisfied.

(1) Defining polynomials f, f' are invertible.

(2) Matrices $A_f, A_{f'}$ of exponents of f and f' are transpose to each other.

(3) f (resp. f') is compactified to a four-termed polynomial F (resp. F') in $|-K_{\mathbb{P}(a)}|$ (resp. $|-K_{\mathbb{P}(b)}|$), where $\mathbb{P}(a)$ (resp. $\mathbb{P}(b)$) is the 3-dimensional weighted projective space whose general members are Gorenstein $K3$ with weight a (resp. b).

In a joint-work with Ueda, the following theorem is proved for every trnaspose-dual pair (B, B') of bimodular singulairites.

Theorem (M-Ueda) *For a transpose-dual pair (B, B'), there exists a reflexive polytope Δ such that $\Delta_F \subset \Delta$ and $\Delta_{F'} \subset \Delta^*$. Here Δ_F (resp. $\Delta_{F'}$) is the Newton polytope of F(resp. F') monomials corresponding to whose lattice points are fixed by an automorphic action of F (resp. F').*

Let Δ be the reflexive polytope obtained in **Theorem** (M-Ueda). For a Δ-regular member S, a natural restriction mapping r from the minimal model $\widetilde{X_\Delta}$ of the toric variety X_Δ associated to Δ to the minimal model $\widetilde{S}$ of S induces a restriction r_* from $H^{1,1}(\widetilde{X_\Delta})$ to $H^{1,1}(\widetilde{S})$. Let $\mathrm{Pic}(\Delta) := H^{1,1}(\widetilde{S}) \cap H^2(\widetilde{S}, \mathbb{Z})$ the Picard lattice of $\widetilde{S}$, and $T(\Delta)$ be its orthogonal complement in the $K3$ lattice. Consider the following problem.

Problem Does an isometry $\mathrm{Pic}(\Delta) \simeq U \oplus T(\Delta^*)$ hold ?

Our main theorem is stated as follows:

Main Theorem *For reflexive polytope Δ, $\mathrm{Pic}(\Delta) \simeq U \oplus T(\Delta^*)$ holds if and only if the map r_* is surjective, where explicit $\mathrm{Pic}(\Delta)$ and $\mathrm{Pic}(\Delta^*)$ are given in the table below. Denote by $C_8^6 := \begin{pmatrix} -4 & 1 \\ 1 & -2 \end{pmatrix}$, and names of singularities follow Arnold.*

Singularity	$\mathrm{Pic}(\Delta)$	$\rho(\Delta)$	$\rho(\Delta^*)$	$\mathrm{Pic}(\Delta^*)$	Singularity
Q_{12}	$U \oplus E_6 \oplus E_8$	16	4	$U \oplus A_2$	E_{18}
$Z_{1,0}$	$U \oplus E_7 \oplus E_8$	17	3	$U \oplus A_1$	E_{19}
E_{20}	$U \oplus E_8^{\oplus 2}$	18	2	U	E_{20}
$Q_{2,0}$	$U \oplus A_6 \oplus E_8$	16	4	$U \oplus C_8^6$	Z_{17}
E_{25}	$U \oplus E_7 \oplus E_8$	17	3	$U \oplus A_1$	Z_{19}
Q_{18}	$U \oplus E_6 \oplus E_8$	16	4	$U \oplus A_2$	E_{30}

Not only the isometry of Picard lattice, but also we find a birational isomorphism between two families.

Corollary *Compactified families of $K3$ surfaces associated to singularities Q_{12} and Q_{18} (resp. $Z_{1,0}$ and E_{25}) have birational general members.*