1. Elastic curves with area constraint

In 1966, M.Kac proposed the following problem: " *Does the complete knowledge of the eigenvalues of the Laplacian determine the shape of the domain?*". Related to this problem, K,Watanabe proposed the variational problem of elastic curve with area constraint, derived the Euler-Lagrange equation and showed the existence of the minimizer by using the direct method. Further, he showed that the minimizer near disk is uniquely determined and the domain is convex.

To determine the minimizer, we need to investigate the Euler-Lagrange equation. The analysis is difficult becaule the Euler-Lagrange equation is nonlinear nonlocal second order boundary value problem.

For this problem, I started joint work with Professors Waichiro Matsumoto and Shoji Yotsutani (Ryukoku University). We obtained representation of all the solutions of Euler-Lagrange equation by using the elliptic functions and complete elliptic integrals and investigated the global structures. My docotr thesis summarizes the basic part of this research. Supervisor is Professor Waichiro Matsumoto.

We showed results for the representation formula in [C2], [C3], [C4], [D1], [D3], [D4]. We reported the summary of the representation formula and global structure in [F1],[F2], e.t.c. We are preparing papers about the global structure.

2. Equilibrium state of elastic ring

In 1976, Tadjbakhsh-Odeh proposed a variational problem of the buckled states of an elastic ring under the uniform pressure. They showed the existence of the minimizer. They also showed that the trivial solution (disk) is minimizer for certain range of pressure. They derived the Euler-Lagrange equation which is nonlinear nonlocal boundary value problem of second-order. In 2009, Takagi-Watanabe obtained a representation all solutions of the Euler-Lagrange equation and showed the partial result of the uniqueness. Later, we obtained other representation formula different from Takagi-Watanabe's ones. They are useful to investigate the global structure. We can completely determined the uniqueness and profiles of the minimizer by using them.

3. The zero points analysis of the homogeneous equation with the complete elliptic integrals

When we investigate the global structure of the solutions of nonlinear boundary value problem, we often need to investigate the zero point of the transcendental equations including the first and second kind complete elliptic integrals. By differentiating the transcendental equation, it often becomes rather complex than original one, since it includes the complete elliptic integrals. We developed a method to overcome this difficulty. For it, we found very accurate approximation formula for E(k)/K(k), which is an application of Gauss's results.