Research plan

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(I) A study of Iwanaga-Gorenstein algebras of finite Cohen-Macaulay type.

An algebra having selfinjective dimension on both sides $d < \infty$ is called Iwanaga-Gorenstein (=IG). The concept of IG algebras is a generalization of that of commutative Gorenstein rings and of selfinjecitve algebras in common. And some experts have studied these IG algebras. However, there are many unknown things in representation theory of IG algebras.

The important property of an IG algebra is the following: the category of Cohen-Macaulay(=CM) modules (which are special modules) forms a Frobenius category, and then its stable category forms a triangulated category. An IG algebra is called *of finite* CM *type* if there are only finitely many isomorphism classes of indecomposable CM modules. The main subject of this project is a construction and a classification of IG algebras of finite CM type. The class of these algebras is the most basic in the class of all IG algebras. This project must be regarded as a generalization of theory of selfinjective algebras due to Tachikawa and Riedtmann.

In the sequel, we consider IG algebras of selfinjective dimension at most $d = 1$.

(I)-1 Construction of IG algebras of finite CM type [7]:

Let A be a hereditary algebra of Dynkin type, C an A-A-bimodule and $\widetilde{A \ltimes C}$ the repetitive algebra of A by C. First, we will investigate the question whether the class of the orbit algebras $\widetilde{A \ltimes C}/G$ of $\widetilde{A \ltimes C}$ for some group G should give a certain class of IG algebras of finite CM type. In the case $C = D(A)$ where $D = \mathrm{Hom}_k(\text{-}, k)$ with the base field k and G is the cyclic group generated by the Nakayama automorphism of $\widetilde{A \ltimes C}$, the algebra $A \ltimes C = \widetilde{A \ltimes C}/G$ is just the trivial extension algebra of A by C, and hence it is a selfinjective algebra. Here, note that $D(A)$ is an injective cogenerator. The concept of cotilting modules is naturally considered as a generalization of that of injective cogenerators. Therefore, we will consider the case that C is a cotilting module such that the endomorphism algebras of C is also isomorphic to A.

(I)-2 Classification of Auslander-Reiten quivers of the category of CM modules over IG algebras of finite CM type:

We consider a certain assumption automatically holds in the case of selfinjective algebras. Under this assumption, the Auslander-Reiten(=AR) quiver of the category of CM modules over an IG algebra of finite CM type is given by the translation quiver $(\mathbb{Z}\Delta/G)_{\mathcal{C}}$ obtained from some Dynkin diagram Δ, the automorphism group G of $\mathbb{Z}\Delta$ and a set $\mathcal{C}$ (*configuration*) of vertices in $\mathbb{Z}\Delta$ similarly to the case of selfinjective algebras. Then, the problem is to characterize the configuration $\mathcal{C}$ in combinatorics. Second, we will consider this problem, considering the case of selfinjective algebras.

(I)-3 Classification of IG algebras of finite CM type:

We mainly consider the *standard* case. An IG algebra is called *standard* if the category of indecomposable CM modules is equivalent to the mesh category of the AR quiver of the category of CM modules. In this case, we will actually calculate algebras from AR quivers which were classified in **(I)-2**. Thus, the third purpose is to investigate whether all standard IG algebras of finite CM type should be given in the way of **(I)-1**. If there is an IG algebra of finite CM type which is not given in the way of **(I)-1**, we will consider the more general construction of IG algebras of finite CM type containing such an algebra. In addition, there must be few non-standard IG algebras of finite CM type because of the result for selfinjective algebras.

(II) A study of Iwanaga-Gorenstein algebras having stable dimension 0.

For an IG algebra, the dimension of the stable category of the category of CM modules is equal to the stable dimension. So, if an IG algebra is of finite CM type, then it has stable dimension 0. Then a natural question arises as to whether the converse should also hold. Hence, the forth purpose is to investigate this question. This is a generalization of my result for selfinjective algebras having stable dimension 0.