Summary of my research

I have studied five topics as follows.

The first is the uniqueness of the direct decomposition of a toric manifold. When we decompose a toric manifold, we can consider direct decompositions as algebraic varieties and as smooth manifolds. I studied both because direct decompositions as algebraic varieties and as smooth manifolds are different. Moreover I proved the direct decomposition of a smooth manifold into copies of complex projective space of complex dimension 1 and simply connected closed smooth 4-manifolds with smooth actions of compact torus action and the direct decomposition of a smooth manifold into copies of real toric manifolds of dimension 1 and 2 are unique.

The second is to construct a topological toric manifold from two fans by gluing complex spaces. A topological toric manifold is a generalization of a toric manifold. A toric manifold is constructed from a fan by several ways (quotient, gluing, embedding in a complex projective space), but in the case of a topological toric manifold only quotient construction is known. I generalized the gluing construction of a toric manifold.

The third is to characterize graphs whose associated toric manifolds admit spin structures by using a necessary and sufficient condition for a toric manifold to admit a spin structure. In this study, a graph may be a pseudograph which may have multiedges and loops. Moreover I characterized building sets whose associated toric manifolds admit spin structures.

The fourth is to study the relation between the set of facet vectors of graph associahedra and root systems. The set of facet vectors of the graph associahedron associated to a simple graph forms a root system if and only if the simple graph is a cycle graph, and the root system associated to the cycle graph is of type A.

The fifth is to describe the representation on the cohomology ring of the toric manifold associated to a simple graph induced by the automorphism group action on the simple graph. I studied cohomology representations associated to special graphs because it is impossible to describe cohomology representations associated to general graphs. I described the cohomology representations when graphs are cycle graphs with 3, 4, and 5 nodes. The automorphism group of a cycle graph is a dihedral group. The reasons why I took cycle graphs are that only cycle graphs appear in the fourth research and that the cohomology representation of the toric manifold associated to type A root system in the different sense from the fourth research is known. Moreover I described the cohomology representation of the toric manifold associated to a graph obtained by removing one edge from a complete graph.