Result

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The clasp number of a knot

The *clasp number* is a knot invariant defined in 1970's, and its research may have not been developed. Professor Kadokami and I attempt to determine the clasp numbers of prime knots with up to 10 crossings. The clasp number $c(K)$ of a knot K is lower bounded by the genus $g(K)$ and the unknotting number $u(K)$, that is, $c(K) \geq g(K)$ or $c(K) \geq u(K)$. Since almost prime knots satisfy $c(K) = g(K)$ or $c(K) = u(K)$, we consider the following question: Does there exist a prime knot K such that $c(K) > g(K)$ and $c(K) > u(K)$. We give an affirmative answer for this question to investigate algebraic properties of the Conway polynomial of a knot K with $c(K) \leq 2$. It is stated that there exist infinitely many prime knots such that $c(K) > g(K)$ and $c(K) > u(K)$. By using this result, we determine the clasp numbers of all but fifteen prime knots with up to 10 crossings.

Diagrams of surface-links and Roseman moves

A *surface-link* is a closed surface embedded in $\mathbb{R}^4$. A *diagram* of a surface-link is its image via a generic projection from $\mathbb{R}^4$ to $\mathbb{R}^3$, equipped with over/under information. D. Roseman showed seven types of local transformations for diagrams, which generate the equivalence relation for surface-links. These seven types of local transformations are called *Roseman moves*, and they are generalization of three types of Reidemeister moves for diagrams of knots. It is well known that each type of Reidemeister moves is independent from the other two types. However, it is known that a certain type, say Type 2, of Roseman moves is not independent from the other six types. I attempt to solve independence problem for Roseman moves, and determine what type except Type 2 independence from the other six types or not. Moreover, jointly working with Professors Tanaka and Oshiro, we construct of two diagrams D and D' representing equivalent surface-links so that any finite sequence between D and D' must contain Roseman moves involving triple points.

Ribbon-clasp surface-links and their presentations

A surface-link is said to be *ribbon* if it bounds a singular handlebodies with only ribbon intersections. It is known that a ribbon surface-link is obtained from a trivial S^2-link by 1-handle surgeries, and that it is in a normal form which is symmetric with respect to $\mathbb{R}^3 \times \{0\}$. Moreover, a ribbon T^2-knot F can be represented by a welded knot K, and the knot group $\pi_1(\mathbb{R}^4 \setminus F)$ is isomorphic to the group $G(K)$ of the welded knot. Professor Kamada and I generalize a clasp intersection in 3-space into that in 4-space and introduce a ribbon-clasp surface-link. A surface-link is said to be *ribbon-clasp* if it bounds a singular handlebodies with only ribbon intersections and clasp ones. Moreover, we prove the following. (1) A ribbon-clasp surface-link is obtained from a trivial S^2-link by 1-handle surgeries or finger moves, and that it is in a normal form which is symmetric with respect to $\mathbb{R}^3 \times \{0\}$. (2) A ribbon-clasp T^2-knot F can be represented by a semi-welded knot K, and the knot group $\pi_1(\mathbb{R}^4 \setminus F)$ is isomorphic to the group $G(K)$ of the semi-welded knot.