Problem 1 We proceed for other transpose-dual pairs to consider the following problem that is stated in the "Abstract of Results" :
let $(\Delta_{MU},\, \Delta'_{MU})$ be a pair of reflexive polytopes obtained in Mase-Ueda's study, which does not attain the lattice mirror symmetry. Is it possible to take another pair $(\Delta,\, \Delta')$ instead of $(\Delta_{MU},\, \Delta'_{MU})$, such that the lattice mirror symmetry holds ?

As is explained, we already have an example with negative answer. However, Problem 1 is also understood from a viewpoint of deformation of singularities. Namely, whether or not it is possible to take a "good" deformation and compactification of singularities. Even if we only get negative answers, this may mean that we can distinguish good from bad singularities in the study of lattice structure of its compactification.

Existence and non-existence of certain curves on $K3$ surfaces are well studied so far. Also, it is important to study whether or not there exists a rational function whose polar is exactly nP, $n \in \mathbb{Z}$ at a point P in a curve. This motivates to define the Weierstrass points. More precisely, for a smooth projective curve C' of genus ≥ 2, a point $P' \in C'$ is called the *Weierstrass point* if $h^0(C',\, \mathcal{O}(gP')) \geq 2$. We call a $K3$ surface X *double sextic* if it is obtained as the minimal resolution of a double covering of $\mathbb{P}^2$ branching along a sextic curve. More generally, some $K3$ surfaces are obtained by double covering some weighted projective plane. We call such a $K3$ surface a *weighted double sextic*.

Problem 2 Characterize weighted double sextic $K3$ surfaces in terms with Weierstrass points of some curves on the $K3$ surface. Conversely, determine what sort of Weierstrass points branch curves of weighted double sextic $K3$ surfaces should have.

Problem 3 Study the moduli space of maps from a $K3$ surface to a Lie group.

As is well known, maps from a circle to a Lie group form a loop group, which is related differential equation. An elliptic $K3$ surface is a surface with generic fibres being elliptic curves over $\mathbb{P}^1$. An elliptic curve is topologically a torus that is a product of two circles. So we are expecting that it is possible to consider a two-dimensional case, that is, maps from a $K3$ surface to a Lie group as a generalization of one-dimensional case, that is maps from a circle to a Lie group like the theory of quantum toroidal algebra that is a generalization of quantum affine algebra in mathematical physics.