Research Results

My main research area is differential geometry, and the keywords are “special Lagrangian submanifold” and “mean curvature flow”. A big motivation of my study is to understand “mirror symmetry” which is a magical phenomenon appearing in various aspects of mathematics and physics. Special Lagrangian submanifolds are one of important subjects to study the mirror symmetry, so I have started with the study of special Lagrangian submanifolds. A special Lagrangian submanifold is defined as a minimal Lagrangian submanifold in a given Calabi-Yau manifold, and it can be written as a solution of a non-linear elliptic PDE locally. Hence it is difficult to construct non-trivial concrete global examples of special Lagrangian submanifolds in a given Calabi-Yau manifold. For instance, in $\mathbb{C}^m$, there are previously known examples constructed by Harvey-Lawson and Joyce. Then I found that their constructions in $\mathbb{C}^m$ work similarly in the cone of a toric Sasaki manifold. Those results are summarized in a paper “Special Lagrangians and Lagrangian self-similar solutions in cones over toric Sasaki manifolds”.

In the study of special Lagrangian submanifolds, it has been learned that it is important to study “Lagrangian mean curvature flow” to get special Lagrangian submanifold more abstractly. A Lagrangian mean curvature flow is an one parameter family of Lagrangian submanifolds whose variation vector fields coincide with its mean curvature vector fields, that is, it satisfies the mean curvature flow equation. Since the volume of the submanifold is decreasing along the mean curvature flow, the Lagrangian mean curvature flow might converge to a special Lagrangian submanifold if it has a long time solution. Concrete examples of Lagrangian mean curvature flows are not so many. Then I generalized the example constructed by Lee-Wang in $\mathbb{C}^m$ to in toric almost Calabi-Yau manifolds in a paper “Weighted Hamiltonian stationary Lagrangian submanifolds and generalized Lagrangian mean curvature flows in toric almost Calabi-Yau manifolds”. These examples of Lagrangian mean curvature flows develop singularities finitely many times and its topologies change when singularities develop.

As indicated by the above examples, Lagrangian mean curvature flows develop singularities, in general. Hence, what we should do is to analyze the asymtotic behavior of a Lagrangian mean curvature flow when it develops singularities. About this problem, in the case that the ambient space is $\mathbb{R}^m$, there is the well-known result due to Huisken, and A. Futaki, K. Hattori and I generalize his result in the case that the ambient space is a Riemannian cone manifold in a paper “Self-similar solutions to the mean curvature flows on Riemannian cone manifolds and special Lagrangians on toric Calabi-Yau cones”. The main theorem states that if a mean curvature flow develops a singularity of type I then its parabolic rescaling converges to a self-shrinker.

Next, in a paper “Ricci-mean curvature flows in gradient shrinking Ricci solitons”, I generalize the result due to Huisken to a Ricci-mean curvature flow coupled with a Ricci flow constructed by a gradient shrinking Ricci soliton. In this paper, a limit of the parabolic rescaling is a self-shrinker defined in a gradient shrinking Ricci soliton, and this definition is a generalization of a self-shrinker in $\mathbb{R}^m$. There are many previously known results about self-similar solutions in $\mathbb{R}^m$. Then, as natural interest, a question that which results already established for self-similar solutions in $\mathbb{R}^m$ also hold for generalized sense? arise. About this question, for example, a lower bound of the diameter of a self-similar solution in $\mathbb{R}^m$ established by Futaki-Li-Li also works for generalized sense, under Lagrangian assumption. It is summarized in a paper “Lagrangian self-similar solutions in gradient shrinking Kähler-Ricci solitons”. In this paper, I also proved that a result of Cao-Li also holds.