Research Plan

(1) (**Invariant Morse functions on torus manifolds**)

Let (M, T) be a torus manifold (i.e., T is the n-dimensional compact torus and M is a $2n$-dimensional closed manifold with a faithful smooth T-action having at least one fixed point). Concerning the existence of invariant Morse functions on a general torus manifold, we conjecture the following:

A torus manifold (M, T) admits a T-invariant Morse function if and only if M admits a T-representation covering.

(For the definition of T-representation covering, we refer to our research result). Since the necessity is already shown in our previous work, the only problem is to show the sufficiency. Although we can state the above conjecture in more general setting, the reason for restricting to this case is the following: in the case of torus manifolds, the orbit space M/T has the structure of a manifold with corners if M admits a T-representation covering. Then it seems that we can construct a T-invariant Morse function on M as the composition of the orbit map $M \to M/T$ and an appropriate Morse function on M/T. One more reason is that the above construction provides a generalization of the construction of invariant Morse functions in the case of symplectic toric manifolds since each fiber of the moment map is a single T-orbit in this case. In the case of a symplectic toric manifold, the height function with respect to a generic element of the Lie algebra provides desired Morse function. It is seemingly crucial to establish a generalization of the height function to the case of orbit space M/T.

(2) (**Extension of the conjecture in (1) to GKM-manifolds**)

We want to solve the GKM-manifold version of the conjecture in (1). In the case of GKM-manifolds, we need to investigate the structure of the orbit spaces of the tangential representations. In there, it perhaps appears a structure which generalizes the structure of manifold with corners. We want to give a precise description of the structure and build up its generalities.

(3) (**GKM theory and Morse theory**)

For general GKM-manifolds, we want to understand the description of the equivariant cohmology in view of Morse theory. In this case an infinite dimensional method might be needed since there are infinitely many GKM-manifolds which never admit invariant Morse functions.