Research Result

(1) (**Intersection of stable and unstable manifolds for invariant Morse-Smale functions**)

To understand GKM-theory in view of Witten's Morse theory, which describes the topology of closed manifolds in terms of negative gradient flows of Morse-Smale functions, we studied the intersection of stable and unstable manifolds of invariant Morse-Smale functions. Let M be a closed manifold and let a compact torus T act on M with finitely many fixed points. Let Φ be a T-invariant Morse-Smale function on M and (p, q) be a pair of critical points of Φ whose Morse indices are differ by 2. Then we proved that if the intersection of unstable manifold of p and the stable manifold of q is non-empty, each connected component of the intersection is equivariantly differmorphic to the open cylinder with canonical S^1-action.

This result seems to suggest that there exists a similarity between Witten's Morse theory and GKM-theory depicted by the following proportion :

$$\begin{aligned}&\text{(Critical point) : (Negative gradient flow)}\\ &= \text{(Fixed point) : (2-sphere).}\end{aligned}$$

(2) (**Alexander polynomials for mixed links**)

A mixed link is the union of finitely many S^1 embedded into the solid torus. We introduced the Alexander polynomial of a mixed link and studied a relationship to the usual Alexander polynomial.

(3) (**Invariant Morse functions and representation coverings**)

To establish the existence theorem of invariant Morse functions on finite dimensional manifolds, we introduced the notion of a representation covering. Let M be a closed manifold and let a compact T act on M with finitely many fixed points. Let M^T be the fixed point set. Then an open covering $\{U_p | p \in M^T\}$ of M is called a T-representation covering if each U_p is T-invariant and T-equivariantly diffeomorphic to the tangential representation T_pM. Then one can show that M admits a T-representation covering if it admits a T-invariant Morse function. In particular, by applying the result to torus manifolds, one finds that every torus manifold which has at least one point whose stabilizer group is non-trivial and finite does not admit invariant Morse function, so we can prove the non-existence of invariant Morse function without using algebraic topology.