Research plan

Geometry, representation theory, and integrable systems arising from Hessenberg varieties

Among several interesting algebraic subsets of the flag variety, there are Springer varieties in geometric representation theory, Peterson varieties in connection with quantum cohomology rings of the flag varieties, and the toric varieties associated with root systems. The *Hessenberg varieties* provides us a unified description of these spaces in the flag variety. In type A_{n-1}, they are defined from an $n \times n$ matrix and a function $h : [n] \to [n]$ satisfying certain properties.

• **Cohomology rings of Hessenberg varieties and representation of symmetric groups** [In collaboration with Mikiya Masuda and Takashi Sato and Tatsuya Horiguchi]

Motivated by the work of Brosnan-Chow, it is important to consider the family of *regular Hessenberg varieties*, and the study of cohomology rings of these Hessenberg varieties are all related to the study of cohomology rings of regular semisimple Hessenberg varieties. In this project, our aim is to provide a nice set of ring generators of the cohomology rings of the regular Hessenberg variety by using GKM theory of its torus equivariant cohomology rings.

• **Weyl character formula for Hessenberg varieties** [In collaboration with Naoki Fujita and Jeremy Lane]

Restricting the Plücker line bundle on the flag variety to a regular semisimple Hessenberg variety, the space of global sections of this line bundle can be seen as a representation of a torus. The character of this representation is described by a natural generalization of the Weyl character formula. We study this representation itself and conditions ensuring vanishing of the higher cohomology groups of this line bundle. This is a first step to a problem whether completely integrable systems (in the sense of symplectic geometry) exists or not on regular semisimple Hessenberg varieties.

• **Toric varieties associated with root systems**

As a special case of the regular semisimple Hessenberg varieties above, we have the toric varieity $X(\Phi)$ associated with a root system Φ. The purpose of this research project is to answer the following question: given root systems Φ_1 and Φ_1, if $X(\Phi_1)$ and $X(\Phi_2)$ are homotopic, are Φ_1 and Φ_2 isomorphic? If the root systems are irreducible of odd rank, then this is true. The case of irreducible root systems of even rank will be the next case.