Research planning

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1. Dynamical system on the sigma divisor integrable by the sigma function

Let $\mathcal{G}$ be the field of meromorphic functions on the 2-th symmetric products of a hyperelliptic curve V of genus 3. In [1], derivations $\mathcal{L}_3^*$ and $\mathcal{L}_5^*$ in $\mathcal{G}$ are defined and dynamical systems on $\mathbb{C}^4$ with coordinates (u_2, u_4, v_5, v_7), which are elements of $\mathcal{G}$, are constructed. Let $\mathcal{F}$ be the field of meromorphic functions on the sigma divisor of V. By using the fact that $\mathcal{F}$ is isomorphic to $\mathcal{G}$, we will construct the derivations L_3^* and L_5^* in $\mathcal{F}$ corresponding to $\mathcal{L}_3^*$ and $\mathcal{L}_5^*$. Since the elements F_2, F_4, F_5, F_7 of $\mathcal{F}$ corresponding to u_2, u_4, v_5, v_7 can be constructed in terms of the sigma function, we will be able to derive dynamical systems on the sigma divisor integrable by the sigma function, which is joint work with V. M. Buchstaber.

2. Inversion of algebraic integrals

Let X be a telescopic curve of genus g associated with a sequence $(a_1, \dots, a_m)$ of positive integers, du the g vector consisting of a basis of holomorphic one forms on X and $\mathrm{Jac}(X)$ the Jacobian variety of X. For the Abel-Jacobi map of one point

$$X \;\to\; \mathrm{Jac}(X), \qquad P = (x_1, \dots, x_m) \;\to\; u := \int_\infty^P du,$$

we want to express the coordinate x_i in terms of u. For hyperelliptic curves and (n, s) curves with small genus, this problem is solved by Onishi and Matsutani [3] by using the sigma function. We will solve the problem for telescopic curves. To do that, it is necessary to derive the vanishing properties of the sigma function on the Abel-Jacobi image. We will derive the vanishing properties of the sigma function by using the series expansion at the origin of the sigma function.

2. Sigma function of arbitrary Riemann surfaces

In [2], the sigma function is generalized to any Riemann surface. The sigma function does not depend on the choice of canonical homology basis (modular invariant). On the other hand the sigma function of (n, s) curves and telescopic curves has stronger algebraic properties than modular invariance, which the coefficients of the series expansion of the sigma function are polynomials of the coefficients of the defining equations. We will express the defining equations of arbitrary algebraic curves by Miura canonical form and show that the coefficients of the series expansion of the sigma funciton of arbitrary Riemann surfaces [2] are polynomials of the coefficients of the defining equations. Because the algebraic curves of genus 1,2 are expressed as hyperelliptic curves, this problem is solved for the case of genus 1,2. The algebraic curves of genus 3 which can not express as telescopic curves are $(3, 5, 7)$ and $(4, 5, 6, 7)$ curves in the framework of Miura. If it is difficult to solve the problem for any Miura canonical form, we will consider the $(3, 5, 7)$ and $(4, 5, 6, 7)$ curves.

References

[1] V. M. Buchstaber and A. V. Mikhailov, "Infinite-Dimensional Lie Algebras Determined by the Space of Symmetric Squares of Hyperelliptic Curves", Funktsional'nyi Analiz i Ego Prilozheniya, Vol. 51, No. 1, pp. 4-27, 2017.

[2] A. Nakayashiki, "Tau Function Approach to Theta Functions", International Mathematics Research Notices, rnv297, (2015).

[3] Y. Ônishi, Complex Multiplication Formulae for Hyperelliptic Curves of Genus Three, *Tokyo J. Math.* , Vol. 21, No. 2, (1998).