Results of research

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（1） Large time behavior of solutions of viscous conservation law with non convex flux

We studied the large-time behavior of the solution to an initial boundary value problem on the half line for scalar conservation law, where the data on the boundary and also at the far field are prescribed. In the case where the flux is convex and the corresponding Riemann problem for the hyperbolic part admits the transonic rarefaction wave (which means its characteristic speed changes the sign), it is known that the solution tends toward a linear superposition of the stationary solution and the rarefaction wave of the hyperbolic part. We proved that even for a quite wide class of flux functions which are not necessarily convex, such the superposition of the stationary solution and the rarefaction wave is asymptotically stable, provided the rarefaction wave is weak. For the proof, L2-weighted energy method plays an important role.

（2） Asymptotic behavior of Damped wave equation on the half line

We studied the asymptotic stability of nonlinear waves for damped wave equations with a convection term on the half line. In the case where the convection term satisfies the convex and sub-characteristic conditions, it is known by the previous works that the solution tends toward a stationary solution. We proved with Professor Y. Ueda that even for a quite wide class of the convection term, linear superposition of the stationary solution and the rarefaction wave is asymptotically stable. Moreover, in the case where the solution tends to the non degenerate stationary wave, we derive that the time convergence rate. We also succeed to remove the ``sub-characteristic condition" which had be always assumed to prove the asymptotic stability of damped wave equation. This work was presented in the keynote lecture at the international congress of Hyp2012. For the proof, L2-weighted energy method play an important role.

（3） Asymptotic behavior of radially symmetric solutions for the Burgers equation

We considered the asymptotic stability and of radially symmetric solutions for Burgers equation on the exterior domain on multi-dimensional space, where the boundary data are prescribed. In the case where the corresponding Riemann problem for the non-viscous part admits the rarefaction wave, we showed that the solution of radially symmetric problem tends toward a linear superposition of the monotone decreasing stationary wave and the rarefaction wave. We also clear that even for the case where the corresponding Riemann problem for the non-viscous part admits the constant state or shock wave, such the superposition are asymptotically stable. Although it seems to be difficult to classify the asymptotic state for the solution of radially symmetric problem, we succeeded to obtain the complete classification of asymptotic state for 3-D case. The proof was given by a standard L2 energy method.