

Feature work

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We will study about region crossing change and related topics as follows.

Topic 1. We are interested in region crossing change on virtual link diagram particularly from the viewpoint of recent works about region crossing change. More precisely, we study projections on surfaces with positive genus.

Topic 2. For a projection G arising from a link diagram D , we define a 1-dimensional simplicial complex $C(G)$ as follows.

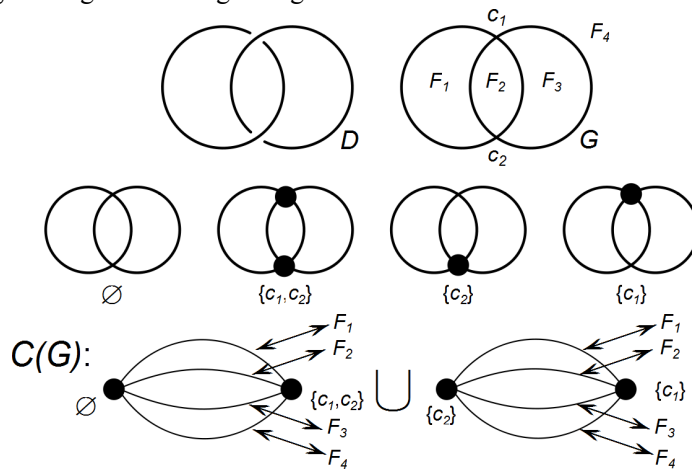
A 0-complex of $C(G)$ corresponds to an element of the power set of vertices of G . There is an edge e such that e connects two vertices v, v' of $C(G)$ if and only if there is a region R such that “the symmetric difference of the set of vertices of G corresponding to v , and the set of vertices of G corresponding to v' ” is changed by the region crossing change at R .

We develop the theory from the viewpoint “the shape of the simplicial complex”. In particular, we give a global understanding of studies about an efficiency of the switching system introduced by Akio Kawauchi-Kengo Kishimoto-Ayaka Shimizu, and Ayumi Inoue-Ryo Shimizu’s results [IS].

Particularly, Kawauchi-Kishimoto-Shimizu

introduced a game based on region crossing change. We would link to introduce the difficulty and the efficiency of the switching system.

Here is an example of the usage the complex: by a result of A. Shimizu, we see that $C(G)$ is connected (in example of the figure, we remark $C(G)$ contains 2 components). On the other hand, we define a simplicial complex $C'(G)$ by region freeze crossing change introduced by Inoue-Shimizu. Then results of Inoue-Shimizu is regarded as “there exists a projection represented of knot such that $C'(G)$ is not connected”. For a mutant of region crossing change, we obtain a simplicial complex from the operation. Then it shows some properties of the operation. Then we develop this philosophy.



Topic 3. A. Shimizu[S] proved that region crossing change is an unknotting operation. Then an invariant called region unknotting number is defined. We will study it. For a projection of a knot on the 2-sphere (or a spherical curve), an operation called “Reidemister move for spherical curve” is introduced. It is important for a theory of singular point that given two spherical curves are translated by certain Reidemister move too. Noboru Ito-Yusuke Takimura-Kouki Tniyama[ITT], Ito- Takimura[IT], Ito[Ito] study this problem with using a concept called “chord diagram”. It will be useful for calculate the region unknotting number.

Reference

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