Research Project

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Let $\mathbb{A}$ be a finite set with $\#\mathbb{A} \geq 2$ and Σ be an infinite set. Given a nonempty set $\Omega \subset \mathbb{A}^{\Sigma}$. Let $\mathbb{N} = \{0, 1, 2, \cdots\}$.

A nonempty closed set $\Theta \subset \mathbb{A}^{\mathbb{N}}$ is called a super-stationary set if for any infinite subset $\mathcal{N} = \{N_0 < N_1 < \cdots\}$ of $\mathbb{N}$, we have $\Theta[\mathcal{N}] = \Theta$. Here, for $\omega \in \mathbb{A}^{\mathbb{N}}$, we define $\omega[\mathcal{N}] \in \mathbb{A}^{\mathbb{N}}$ as $\omega[\mathcal{N}](n) = \omega(N_n)$ $(\forall n \in \mathbb{N})$ and $\Theta[\mathcal{N}] = \{\omega[\mathcal{N}];\ \omega \in \Theta\}$. We know some characterizations of super-stationary sets. One of them is that it is written as $\mathcal{P}(\Xi)$ with a finite set $\Xi \subset \mathbb{A}^{+}$ of prohibited words satisfying the condition (#) (refer [56]). Here, $\mathbb{A}^{+} = \cup_{k=1}^{\infty} \mathbb{A}^k$ and $\xi \in \mathbb{A}^k$ is said to be prohibited in $\omega \in \mathbb{A}^{\mathbb{N}}$ if $\omega(s_1) \cdots \omega(s_k) = \xi$ does not hold for any $\{s_1 < \cdots < s_k\} \subset \mathbb{N}$, and $\mathcal{P}(\Xi)$ is the set of $\omega \in \mathbb{A}^{\mathbb{N}}$ such that any word in $\xi \in \Xi$ is prohibited.

Considering Σ as a discrete space, let $\beta\Sigma$ be the Stone-Cech compactification of it. That is, $\beta\Sigma$ is the set of ultra-filters on Σ. Here, a principal ultra-filter is identified with an element of Σ, so that $\beta\Sigma \setminus \Sigma$ is the set of non-principal ultra-filters on Σ. For $\chi_i \in \beta\Sigma_i$ $(i = 1, 2)$, let $\chi_1 \times \chi_2 \in \beta(\Sigma_1 \times \Sigma_2)$ be such that

$$\chi_1 \times \chi_2 = \{U \subset \Sigma_1 \times \Sigma_2;\ \{x \in \Sigma_1;\ \{y \in \Sigma_2;\ (x, y) \in U\} \in \chi_2\} \in \chi_1\}.$$

Hence, for $\chi \in \beta\Sigma$, $\chi^k \in \beta(\Sigma^k)$ $(k = 1, 2, \cdots)$ can be defined inductively.

For the above Ω and $S = (s_1, \cdots, s_k) \in \Sigma^k$, define $\Omega[S] \subset \mathbb{A}^k$ by $\Omega[S] = \{\omega(s_1) \cdots \omega(s_k) \in \mathbb{A}^k;\ \omega \in \Omega\}$. Let $\mathcal{B}(\mathbb{A}^k)$ be the family of subsets of $\mathbb{A}^k$. Then, the mapping $S \mapsto \Omega[S]$ from Σ^k to $\mathcal{B}(\mathbb{A}^k)$ can be uniquely extended to a continuous mapping from $\beta(\Sigma^k)$ to $\mathcal{B}(\mathbb{A}^k)$. Hence for $\chi \in \beta\Sigma$, $\Omega[\chi^k]$ makes sense as the value at χ^k of the extended mapping. In fact, for $\Lambda \subset \mathbb{A}^k$, $\Omega[\chi^k] = \Lambda$ implies that $\{S \in \Sigma^k;\ \Omega[S] = \Lambda\} \in \chi^k$. For any $\chi \in \beta\Sigma$, $\Omega[\chi^k]$ $(k = 1, 2, \cdots)$ is a consistent family of subsets of $\mathbb{A}^k$, and $\Omega[\chi^\infty] \subset \mathbb{A}^{\mathbb{N}}$ is determined as the projective limit. Then, it is known [61] that

"$\Omega[\chi^\infty]$ is always a super-stationary set, which we call a super-stationary factor of Ω. If χ is principal, that is, $\chi \in \Sigma$, then $\Omega[\chi^\infty] = \{a^\infty; a \in \Omega[\chi]\}$, where $a^\infty = aaa\cdots$. On the other hand, for any $\chi \in \beta\mathbb{N} \setminus \mathbb{N}$ and a nonempty set $\Theta \subset \mathbb{A}^{\mathbb{N}}$, Θ is a super-stationary set if and only if $\Theta[\chi^\infty] = \Theta$."

In [61], I obtained the super-stationary factors for various symbolic dynamics (Ω, T). I want to study the meaning of them as dynamical system. If T is considered as the unit time lapse, then the super-stationary factors represent properties of the dynamical system depending only on the time order, but not on the quantity of time. These properties are interesting since they are completely oposit to what have been interested in so far, that is, properties which are sensitive to time scaling like entropy.

I'm also interested in obtaining a single valued criterion of randomness for finite sequences of symbols. The algorithmic complexity defined by Kolmogorov and Chaitin is such a criterion which is theoretically perfect, but it has two shortcomings. First is that it is not a computable function, second is that it has an ambiguity up to adding an arbitrary constant, so that it is not at all practical. In [64], I proposed a new criterion which is enough satisfactory both theoretically and practically. I'll try to develope this work.

In 2017, I'm planning to visit Beihang University and Huazhong University of Science and Technology in China for a few months for collaborations in dynamical systems and fractal geometry.