Plan

Kengo Kawamura

A *surface-knot* is a connected closed surface embedded in $\mathbb{R}^4$, which is a generalization of a knot as a connected closed curve embedded in $\mathbb{R}^3$. Considering a knot as the image of a generic map from a connected closed curve into $\mathbb{R}^3$, its generalization is the image of a generic map from a connected closed surface into $\mathbb{R}^4$, which is a *singular surface-knot*. In other words, a singular surface-knot is a connected closed surface imemrsed in $\mathbb{R}^4$ such that each multiple point is a transverse double point called a *node*. Therefore, it is also important to investigate a singular surface-knot. The purpose of my research is to study a singular surface-knot in terms of diagram representation. The main research is the following.

(1) Calculating a new quandle homology group and its application

A *quandle* is a non-empty set with a binary operation whose conditions derive from Reidemeister moves, and it is compatible with classical knots and surface-knots. In the late 1990's, Carter et al. introduced quandle homology theory. For a diagram of a surface-knot, a state sum associated with a 3-cocycle of quandle homology can be defined. The state sum is an invariant of a surfaec-knot, which is called the *quandle cocycle invariant*. It is known that the quandle cocycle invariant is effective to estimate the minimal triple point number of a surface-knot and to evaluate an invertibility of a surface-knot. The purpose of my research is to investigate geometrical properties of a singular surface-knot in terms of diagram representation. In the last year, I introduced quandle homology closely related to diagrams of singular surface-knots. For a diagram of a singular surface-knot, a state sum associated with a 3-cocycle of the new quandle homology can be similarly defined. (The state sum may be an invariant of a singular surface-knot.) I am planning to calculate new quandle homology groups and non-trivial 3-cocycles by using a computer, and investigate geometrical properties of a diagram of a singular surface-knot as application.

(2) Local moves on singular surface-knot diagrams which generate the equivalence relation of singular surface-knots

Seven types of local moves, called *Roseman moves*, are crucial to study surface-knots in terms of diagrams. The reason why is that two surface-knots are equivalent if and only if their diagrams are related by a finite sequence of Roseman moves. That is, Roseman moves generate the equivalence relation of surface-knots. Considering a singular surface-knot and its diagram, local moves related to a node are needed to give an analogous result. There is PN-move as a candidate of the local move, where PN means that passing a node through a sheet. It is conjectured that local moves on singular surface-knot diagrams which generate the equivalence relation of singular surface-knots are Roseman moves and PN-moves. I am planning to solve this conjecture.