Result

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(1) The clasp number of a knot

The *clasp number* is a knot invariant defined in 1970's, and its research may have not been developed. Professor Kadokami and I attempt to determine the clasp numbers of prime knots with up to 10 crossings. To determine the clasp number, we usually use a lower bound by the genus or the unknotting number. We focus on the Conway polynomial of knots to break the case that we can not determine the clasp numbers in such a way. We prove that there exist infinitely many prime knots which can not be determined the clasp numbers in such a way by investigating algebraic properties of the Conway polynomial of a knot whose clasp number is less than or equal to two.

(2) Diagrams of surface-links and Roseman moves

A *surface-link* is a closed surface embedded in $\mathbb{R}^4$. A *diagram* of a surface-link is its image via a generic projection from $\mathbb{R}^4$ to $\mathbb{R}^3$, equipped with over/under information. Roseman showed seven types of local transformations (called *Roseman moves*) for diagrams, which generate the equivalence relation for surface-links. We solve independence problem for Roseman moves by investigating geometrical information of surface-link diagrams. Moreover, jointly working with Professors Tanaka and Oshiro, we construct of two diagrams D and D' presenting equivalent surface-links so that any finite sequence between D and D' must contain Roseman moves involving triple points.

(3) Ribbon-clasp surface-links

A surface-link is said to be *ribbon* if it is the boundary of a singular handlebodies with only ribbon intersections. Professor Kamada and I generalize a clasp intersection in 3-space into that in 4-space and introduce a ribbon-clasp surface-link. A surface-link is said to be *ribbon-clasp* if it is the boundary of a singular handlebodies with only ribbon intersections and clasp intersections. Moreover, we prove that analogies of geometrical properties for ribbon surface-links hold for ribbon-clasp surface-links.

(4) Quandle and singular surface-knots

Up to now, some variations of quandle homology which are introduced in the late 1990's are constructed. For a surface-knot diagram, a state sum associated with a 3-cocycle of a usual quandle homology can be calculated. Since this is invariant under Roseman moves, it is an invariant of a surface-knot (called the *quandle cocycle invariant*). We construct a new variation of quandle homology to introduce the quandle cocycle invariant for singular surface-knots. For a singular surface-knot diagram, a state sum associated with a quandle 3-cocycle in a usual sense does not become an invariant of a singular surface-knot. However, we can obtain a state sum which is invariant under PN-move and Roseman moves by using a quandle 3-cocycle in a new sense. Moreover, we give a sufficient condition which a singular surface-knot is 'S^2_*-irreducible', and construct some S^2_*-irreducible singular surface-knots.