Problem 1 (1) In [1], we have stated a conjecture :
For each of the following transpose-dual pairs $(Z_{13},\ J_{3,0}),\ (Z_{17},\ Q_{2,0}),\ (U_{16},\ U_{16}),\ (W_{17},\ S_{1,0}),\ (W_{18},\ W_{18}),\ (S_{17},\ X_{2,0})$, there DOES not exist compactifications F, F' of defining polynomial and reflexive polytopes Δ and Δ' such that

$$(**) \qquad \Delta^* \simeq \Delta',\ \Delta_F \subset \Delta,\ \Delta_{F'} \subset \Delta', \quad \textit{and} \quad \operatorname{rk} L_0(\Delta) = 0$$

hold. Moreover, $\rho(\Delta) + \rho(\Delta') = 20$ NEVER happens.

The first problem is to determine wether or not this conjecture is true. Also, we would like to seek another duality of lattices : for instance, Rohsiepe's duality [2].

(2) As the second part of the first problem, we determine whther or not the following conjecture is true: For each of the dual pairs $(Z_{1,0},\ Z_{1,0})$, $(U_{1,0},\ U_{1,0})$, $(Q_{17},\ Z_{2,0})$, $(W_{1,0},\ W_{1,0})$, the pair of families $(\mathcal{F}_\Delta,\ \mathcal{F}'_{\Delta'})$ obtained in the paper [1] satisfies the relation

$$(\sharp) \qquad \operatorname{Pic}(\Delta)^{\perp}_{\Lambda_{K3}} \simeq U \oplus \operatorname{Pic}(\Delta').$$

Problem 2 As a generalisation of a construction of a $K3$ surface as a (minimal model of) the double covering of the projective plane $\mathbb{P}^2$ branching along a smooth sextic curve B, a $K3$ surface is constructed as a (minimal model of) the double covering of the weighted projective planes $\mathbb{P}$ of weights $(1,1,4)$, $(1,3,8)$, and $(1,4,5)$.

Our second problem is to characterize weighted double sextic $K3$ surfaces in terms with Weierstrass semi-groups of points of smooth curves on the $K3$ surface. Conversely, determine what sort of Weierstrass points, branch curves of weighted double sextic $K3$ surfaces should have.

Problem 3 Study the moduli space of maps from a $K3$ surface to a Lie group.

As is well known, maps from a circle to a Lie group form a loop group, which is related differential equation. An elliptic $K3$ surface is a surface with generic fibres being elliptic curves over $\mathbb{P}^1$. An elliptic curve is topologically a torus that is a product of two circles. So we are expecting that it is possible to consider a two-dimensional case, that is, maps from a $K3$ surface to a Lie group as a generalization of one-dimensional case, that is maps from a circle to a Lie group.

As to strategy, we have some ideas : 1) before elliptic $K3$ surfaces, we should first study *rational* elliptic surfaces, for which a lot of studies are done; 2) we should study actions of Lie groups (of infinite-dimensional) on $K3$ surfaces (if there exists).

References

[1] MASE, M. Polytope duality for families of $K3$ surfaces associated to transpose duality. *to appear in Commentalli Math. St. Pauli.*.

[2] ROHSIEPE, F. Lattice polarized toric $K3$ surfaces. *arXive:hep-th/0409290v1* (2004).