1. elastic curves with area constraint

In 1966, M.Kac proposed the following problem: " *Does the complete knowledge of the eigenvalues of the Laplacian determine the shape of the domain?*". In the relation to this problem, K,Watanabe proposed the variational problem of elastic curve with area constraint, derived the Euler-Lagrange equation and showed the existence of the minimizer by using the direct method. Further, he showed minimizer near disk which is uniqunely detemined and its domain is convex.

To determine the minimizer, we need to investigate the Euler-Lagrange equation. The analysis is difficult becaule the Euler-Lagrange equation is nonliner nonlocal second order boundary value problem.

For this problem, I started joint work with Professor.Waichiro Matsumoto and Professor.Shoji Yotsutaniof Ryukoku University. We represented all the solutions of Euler-Lagrange equation by using the elliptic functions and complete elliptic integrals and investigated the global strucuture. My docotr thesis summarizes the basic part of this research. Supervisor is Professor Waichiro Matsumoto of Ryukoku University.

We showed the result for the reprasentation formula in [C2], [C3], [C4], [D1], [D3], [D4]. The summary of the representation formula and global structure, we reported in [F1],[F2], ect. We prepare to post thesis for the global structure.

2. Equilibrium state of elastic ring

In 1976, Tadjbakhsh-Odeh proposed the variational problem of the buckled states of an elastic ring under the uniform pressure. They showed the existence of the minimizer. They also showed that the trivial solution (disk) is minimizer for certain range of pressure. They derived the Euler-Lagrange equation which is nonlinear nonlocal boundary value problem of second-order. In 2009, Takagi-Watanabe represented all the solutions of Euler-Lagrange equation and showed the partial result of the uniqueness of the minimizer which included the the non-trivial solution. On the other hand, we showed the another representation formula different from Takagi-Watanabe's ones. They are useful to investigate the global structure. We can completely determined the uniqueness of the minimizer by using them.

3. The zero points analysis of the homogeneous equation with the complete elliptic integrals

When we investigate the global structure of the solutions of nonlinear boundary value problem, we often need to investigate the zero point of the transcendental equations including the first and second kind complete elliptic integrals. By differentiating the transcendental equation, it often becomes rather complex than original one, since it includes the complete elliptic integrals. Thus we constructed the approximation formula of E(k)/K(k).