

RESEARCH PROGRAMME

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Based on the research results, we would like to work on the following three issues.

Weakly reflective submanifolds We want to classify weakly reflective orbits of commutative Hermann actions. There exists an austere submanifold in hypersphere which is not weakly reflective. Thus, we can deduce that there exists an austere orbit of a Hermann action which is not weakly reflective. To prove that a submanifold is not weakly reflective, we need to give a necessary condition for submanifolds to be weakly reflective. In hypersurface cases, there is a useful necessary condition. However, in higher codimension cases, we have not found such a necessary condition. If we found necessary conditions, then we can prove that submanifolds are not weakly reflective. The method of previous studies was the method for constructing reflections using inner automorphisms. We would like to consider how to construct reflections using outer automorphisms.

Recently, Y. Taketomi introduced the notion of arid submanifolds which is a generalization of weakly reflective submanifolds. An arid submanifold is minimal. Thus, by investigating arid submanifolds it is expected to find new properties of weakly reflective submanifolds.

Correspondence of geometric properties of orbits of Lie group action Let (G, K_1, K_2) be a commutative symmetric triad. The three Lie group actions $K_2 \curvearrowright G/K_2$, $K_1 \curvearrowright G/K_1$ and $K_2 \times K_1 \curvearrowright G$ have the same orbit space. Moreover, Geometric properties of orbits of these actions correspond. For example, minimal orbits and austere orbits of the three actions correspond, respectively. On the other hand, totally geodesic orbits of the actions $K_2 \curvearrowright G/K_2$ and $K_1 \curvearrowright G/K_1$ correspond, but totally geodesic orbit of $K_2 \times K_1 \curvearrowright G$ does not correspond. We would like to consider such as correspondence for weakly reflective orbits and biharmonic orbits.

Conversely, we would like to consider “when do geometric properties correspond?”. For Iwasawa decomposition $G = KAN$, we investigate correspondence of properties of orbits the three actions $K \curvearrowright G/N$, $N \curvearrowright G/K$ and $K \times N \curvearrowright G$.

Biharmonic submanifolds We can apply our method to higher codimension cases. We can expect that we obtain many examples of higher codimension biharmonic submanifolds using this method. In fact, several biharmonic submanifolds of higher codimension can be constructed in this manner. Also, for orbits of the action $K_2 \times K_1 \curvearrowright G$, we can characterize the biharmonic condition in terms of symmetric triads, and we can expect that we obtain many examples of biharmonic submanifolds in compact Lie groups. Moreover, we can apply our method to orbits of the isotropy representation of symmetric spaces. Since examples of the biharmonic submanifold of higher codimension are not much found, if we can get a biharmonic submanifold of higher codimension in the sphere, it is expected to help the study of biharmonic maps.

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