

# Results of research

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We consider  $\beta$ -deformation of the quiver matrix models in the light of the recent progress on 2d-4d connection of conformal field theories, in particular, the Alday-Gaiotto-Tachikawa (AGT) conjecture. A quantum spectral curve is introduced in the case of  $SU(n)$  quiver matrix model. Residue analysis is provided both for the curve of the  $SU(n)$  matrix model with the “multi-log” potential and for the Seiberg-Witten curve in the case of  $SU(n)$  with  $N_f = 2n$  flavors, leading to the matching of the mass parameters. The isomorphism of two curves is made manifest (Ref.[27] of the Publication List).

We observe that the Dotsenko-Fateev integral representation of the conformal block of 2d conformal field theory can be interpreted as a  $\beta$ -deformed matrix model of Selberg type. Using the formula associated with the Jack polynomials, we established the method of generating  $q$ -expansion coefficients for conformal block and Nekrasov function for  $\mathcal{N} = 2$  supersymmetric  $SU(2)$  gauge theory with four flavors ([28]).

We then consider a series of massive scaling limits of the  $\beta$ -deformed matrix model of Selberg type ( $SU(2)$  with  $N_f = 4$ ) which reduce the number of flavors to  $N_f = 3$  and subsequently to  $N_f = 2$  ([29]).

We consider  $\beta$ -deformed quiver matrix model based on the affine Lie algebra  $A_n^{(1)}$ . The Virasoro constraint of this model is determined. For  $n = 1, 2$  cases, the explicit forms of the loop equations are obtained ([30]).

The (W)AGT conjecture implies there is a correspondence between the partition functions of the four-dimensional  $\mathcal{N} = 2$  supersymmetric gauge theories and the conformal blocks of the two-dimensional theories with the Virasoro or W symmetries. The “ $q$ -deformed” version of (W)AGT conjecture states that the  $q$ -lifted version of the partition function of five-dimensional gauge theories and the “conformal blocks” of the two-dimensional theories with the  $q$ -deformed Virasoro/W symmetries. Starting from this  $q$ -version of (W)AGT conjecture, we demonstrate by taking a certain  $r$ -th root of unity limit in  $q$ , the correspondence between the four-dimensional partition function on the ALE space of A-type and the conformal blocks of the two-dimensional theories with the super-Virasoro symmetry or its generalization is automatically generated ([32]). Furthermore, we demonstrated that the parafermions appear in the  $r$ -th root of limit of the  $q$ -deformed Virasoro and the  $q$ -deformed W algebra ([34]).

A  $q$ -deformed vertex operator is determined from the five-dimensional  $SU(2)$  Nekrasov partition function based on the  $q$ -AGT conjecture. We obtained a  $q$ -deformed version of Coulomb gas representation of the conformal block by using the vertex operators and  $q$ -screening charges. After slightly changing the position of one of the vertex operators, we have checked that the  $q$ -block coincides with 5D Nekrasov function in low degrees of instanton expansion ([35]).

A set of Schwinger-Dyson equations for the resolvents are considered in a class of supersymmetric Chern-Simons-matter matrix models. In the planar limit, these loop equations reduce to two independent algebraic cubic equations for the two planar resolvents ([36]).