

Brief summary on the study

The author has studied on the elliptic Ruijsenaars model and the elliptic Ding-Iohara-Miki algebra. First we give backgrounds. In classical cases as well as quantum cases, many-body problems are generally complicated, however, there exists the elliptic Ruijsenaars model which is solvable quantum many-body system by means of Liouville. We call the Hamiltonian for the model the elliptic Ruijsenaars operator. Then, the q -difference operator contains the theta function, thus we may understand that the elliptic Ruijsenaars model relates to some elliptic functions. We may consider the trigonometric limit of the elliptic Ruijsenaars operator, and the limit is essentially the same as the Macdonald q -difference operator. Then the following facts are known:

- Eigenfunctions for the trigonometric Ruijsenaars model are given by the Macdonald polynomials.
- From the free field realization of the Macdonald operator, the Ding-Iohara-Miki algebra which is a kind of quantum group arises.

On the other hand, to determine eigenfunctions for the elliptic Ruijsenaars operator is still open. Thus it is natural to have a question; "Can we obtain something like a representation of the Ding-Iohara-Miki algebra from the elliptic Ruijsenaars model?" On the problem, the author has shown the followings.

- The elliptic Ruijsenaars operator is realized by boson operators successfully.
- The elliptic Ding-Iohara-Miki algebra which is an elliptic analog of the Ding-Iohara-Miki algebra is obtained.

The above results can be represented by:

$$\begin{array}{ccc}
 \text{Elliptic Ruijsenaars operator} & \xrightarrow{\text{free field realization}} & \text{Elliptic Ding-Iohara-Miki algebra} \\
 \uparrow \text{elliptic deformation} & & \uparrow \text{elliptic deformation} \\
 \text{Macdonald operator} & \xrightarrow{\text{free field realization}} & \text{Ding-Iohara-Miki algebra}
 \end{array}$$

On theoretical physics side, it has been considered that studies on representations of the Ding-Iohara-Miki algebra are important to understand the 5-dimensional AGT conjecture. Thus it is probable that the elliptic Ding-Iohara-Miki algebra has relations to the 6-dimensional AGT conjecture. In addition, the free field realization technique introduced by the author is also used by Iqbal-Kozcaz-Yau, Nieri to construct the representation of an elliptic analog of the q -Virasoro algebra. The author believes that more materials related to the elliptic Ruijsenaars model or the elliptic Ding-Iohara-Miki algebra will occur in the future.