Research Results

My main research area is differential geometry, and its keywords are "special Lagrangian submanifold" and "mean curvature flow". These two are important subjects to study the mirror symmetry. A special Lagrangian submanifold is defined as a minimal Lagrangian submanifold in a given Calabi-Yau manifold, and it can be written as a solution of a non-linear elliptic PDE locally. Hence it is difficult to construct non-trivial concrete global examples of special Lagrangian submanifolds in a given Calabi-Yau manifold. For instance, in $\mathbb{C}^m$, there are previously known examples constructed by Harvey-Lawson and Joyce. In [3], I generalized their constructions in the cone of a toric Sasaki manifold.

Lagrangian mean curvature flows are important to get special Lagrangian submanifold abstractly. Since the volume of the submanifold is decreasing along the mean curvature flow, the Lagrangian mean curvature flow might converge to a special Lagrangian submanifold if it has a long time solution with bounded curvature. In [2], I generalized the example constructed by Lee-Wang in $\mathbb{C}^m$ in toric almost Calabi-Yau manifolds. These examples of Lagrangian mean curvature flows develop singularities finitely many times and its topologies change when singularities occur.

As indicated by the above examples, Lagrangian mean curvature flows develop singularities, in general. Hence, what we should do is to analyze the asymtotic behavior of a Lagrangian mean curvature flow when it develops singularities. About this problem, in the case that the ambient space is $\mathbb{R}^m$, there is the well-known result due to Huisken, and Futaki, Hattori and I generalized his result in the case that the ambient space is a Riemannian cone manifold in [1]. The main theorem states that if a mean curvature flow develops a singularity of type I then its parabolic rescaling converges to a self-shrinker. In [5], I further generalized these results to a coupled flow of a Ricci flow (constructed by a gradient shrinking Ricci soliton) and a mean curvature flow.

There are many known results about self-similar solutions in $\mathbb{R}^m$. Then, as natural interest, we can check that which results for self-similar solutions in $\mathbb{R}^m$ also hold for generalized self-similar solutions in gradient shrinking Ricci solitons in sense of [5]. Then, I generalized a result of Futaki-Li-Li which gives a lower bound of the diameter of a self-similar solution in $\mathbb{R}^m$ in [4]. In this paper, I also generalized a result of Cao-Li.

In [5], I considered a coupled flow of a Ricci flow and a mean curvature flow. It is called a Ricci-mean curvature flow. In [6], I constructed examples of Ricci mean curvature flows. Its ambient space is a gradient shrinking Kähler Ricci soliton on a total space of $\mathbb{P}^1$-fibration over a projective space given by Cao and Koiso. For this ambient space, I investigated the motion of a lens space under a Ricci-mean curvature flow, and proved that if the initial radius of the lens space is lager than a value then it collapses to ∞-section and if the initial radius is smaller than a value then it collapses to 0-section. This gives a first non-trivial example of Ricci-mean curvature flows.

References

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