

Research Plan

(1) (Invariant Morse functions on GKM-manifolds)

Let (M, T) be a GKM-manifold. Concerning the existence of invariant Morse functions on a general GKM-manifold, I conjecture the following:

A GKM manifold (M, T) admits a T -invariant Morse function if and only if M admits a T -representation covering.

(For the definition of T -representation covering, we refer to our research result). Since the necessity is already shown in our previous work, the only problem is to show the sufficiency. Although we can state the above conjecture in more general setting, the reason for restricting to this case is that by the result explained in research result (the structure of the Hessian matrix of invariant functions on GKM-representation spaces), in the case of GKM-manifold one can easily check the non-degeneracy of invariant functions.

To construct invariant Morse functions, I will examine the usage of a variant of moment maps arising from equivariant line bundle over GKM-manifolds. By combing the previous result stated in research result, I will construct invariant Morse functions over GKM-manifolds.

(2) (Orbit space of GKM-manifolds)

For torus manifolds it is useful to focus on a certain combinatorial structure arising from orbit spaces. I want to generalize this kind of phenomenon to the case of GKM-manifolds. In there, it perhaps appears a structure which generalizes the structure of manifold with corners. I want to give a precise description of the structure and build up its generalities.

(3) (GKM theory and Morse theory)

For general GKM-manifolds, we want to understand the description of the equivariant cohomology in view of Morse theory. In this case an infinite dimensional method might be needed since there are infinitely many GKM-manifolds which never admit invariant Morse functions.