

Research Result

(1) **(Intersection of stable and unstable manifolds for invariant Morse-Smale functions)**

To understand GKM-theory in view of Witten's Morse theory, which describes the topology of closed manifolds in terms of negative gradient flows of Morse-Smale functions, we studied the intersection of stable and unstable manifolds of invariant Morse-Smale functions. Let M be a closed manifold and let a compact torus T act on M with finitely many fixed points. Let Φ be a T -invariant Morse-Smale function on M and (p, q) be a pair of critical points of Φ whose Morse indices are differ by 2. Then we proved that if the intersection of unstable manifold of p and the stable manifold of q is non-empty, each connected component of the intersection is equivariantly diffeomorphic to the open cylinder with canonical S^1 -action.

(2) **(Alexander polynomials for mixed links)**

A mixed link is the union of finitely many S^1 embedded into the solid torus. We introduced the Alexander polynomial of a mixed link and studied a relationship to the usual Alexander polynomial.

(3) **(Invariant Morse functions and representation coverings)**

To establish the existence theorem of invariant Morse functions on finite dimensional manifolds, we introduced the notion of a representation covering. Let M be a closed manifold and let a compact T act on M with finitely many fixed points. Let M^T be the fixed point set. Then an open covering $\{U_p | p \in M^T\}$ of M is called a T -representation covering if each U_p is T -invariant and T -equivariantly diffeomorphic to the tangential representation $T_p M$. Then one can show that M admits a T -representation covering if it admits a T -invariant Morse function. In particular, by applying the result to torus manifolds, one finds an infinite family of torus manifolds so that each member never admit invariant Morse function.

(4) **(Invariant functions on GKM-representation spaces)**

I investigated the structure of the Hessian matrix of invariant functions on GKM-representation spaces at the origin, with respect to the coordinate arising from the irreducible representation, and proved that it is of diagonal type. This phenomenon is the same to the case of Morse normal form of Morse functions, and reveals a Morse theoretic meaning of the GKM-condition.