# Research Plan

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Under my previous research of the transverse instability I study the following contents.

**(1) Transverse instability of various equation**

In my plan of the research, it is important to study the transverse instability on various equations. There is the relation between the difficulty of problems and treated standing waves to investigate the asymptotic behavior of solutions near by standing waves. Studying the stability and the linearized operator of standing waves on various equations, we find a suitable standing wave and a suitable equation to analyze the dynamics of solutions of the equation.

In the previous result, studying the behaver of solutions around the center manifolds precisely, Comech and Pelinovsky showed the instability of a standing wave with the degenerate linearized operator. Since the degeneracy of the linearized operator in the our case is worse, we can not apply the argument by Comech and Pelinovsky to show the instability in the critical case. However, there is the possibility which we can show the instability in the critical case, if we combine between the analysis of the bifurcation and the argument. If we show the instability in the critical case by analyzing the behaver of solutions around the center manifolds, there is the possibility to show the behaver of solutions apart from the unstable standing wave.

**(2) The existence of the center stable manifolds**

It is conjectured that solutions near by unstable solitary waves decompose some stable solitary waves and the the scattering part in many cases. This conjecture is true for some integrable equation, however it is very difficult to prove this conjecture for (ZK) which is not integrable. For the first step to approach solving this conjecture, I study the center stable manifolds near by unstable line solitary waves of (ZK) on $\mathbb{R} \times \mathbb{T}_L$.

To construct the center stable manifolds, I apply the argument by Nakanishi–Schlag'12 for the construction of the center stable manifolds in the sense the orbital stability. After that, applying the argument by Martel–Merle, I show the solution on this center stable manifolds tends to line solitary waves.

**(3) The stability for multi-solitary waves of KP-I and ZK**

In some numerical results, it was showed that a solution with an initial data near the unstable line solitary wave splits into two solitary waves which are a line solitary wave and a non-line solitary wave. This result implies the possibility of the existence of such a multi-solitary wave apart from a unstable line solitary wave. Thus, I study the existence of such multi-solitary waves.

From the result (iii), I get the asymptotic stability of the non-line solitary waves which is connected with the branch of line solitary waves. Applying the Martel–Merle argument, I study the existence and the asymptotic stability of multi-solitary waves. Since the group velocity of KP-I equation is not sign-definite, it is difficult to prove the Liouville type theorem for KP-I equation. Thus, for line solitary waves of KP-I equation, I study the orbital stability.

**(4) The asymptotic behavior of solutions apart from unstable solitary waves**

From the result (iii), if we neglect small solitary waves, the solution apart from a unstable line solitary wave tends to a solitary wave. However, we do not know which solitary wave the solution gets closer to after the solution exits a neighborhood of the line solitary wave. Investigating the property of the unstable manifolds, the energy structure around the bifurcation point and the virial type estimate, I study the precise behavior of the solution apart from line solitary waves.