Research results

The geometry and topology of Hessenberg varieties ([1.1], [1.2], [1.3], [1.4], [1.5], [2.1] in the List)

• [Joint work with Megumi Harada, Tatsuya Horiguchi, and Mikiya Masuda] We gave an explicit presentation of the cohomology ring $H^*(\mathrm{Hess}(N,h);\mathbb{Q})$ of the regular nilpotent Hessenberg variety $\mathrm{Hess}(N,h)$, and by considering the $\mathfrak{S}_n$-representation on the cohomology ring of regular semisimple Hessenberg variety $\mathrm{Hess}(S,h)$, we obtained a ring isomorphism $H^*(\mathrm{Hess}(N,h);\mathbb{Q}) \cong H^*(\mathrm{Hess}(S,h);\mathbb{Q})^{\mathfrak{S}_n}$.

• [Joint work with Peter Crooks] The main interest of this project is the Hessenberg variety over the minimal nilpotent orbit. We described the Euler number, the irreducible components, and a presentation of the cohomology ring.

• [Joint work with Lauren DeDieu, Federico Galetto, and Megumi Harada] We showed that the regular nilpotent Hessenberg variety $\mathrm{Hess}(N,h)$ in type A is a local complete intersection. We also studied a flat family of $\mathrm{Hess}(S,h)$ degenerating to $\mathrm{Hess}(N,h)$, and we computed the degree of their projective embeddings as an application. We also explicitly determined the Newton-Okounkov body of the Peterson variety of type A_2 with respect to a choice of a valuation on the field of rational functions.

• [Joint work with Mikiya Masuda and Tatsuya Horiguchi] For the case that $h = (h(1), n, \ldots, n)$, we explicitly determined the ring structure of the cohomology of the regular semisimple Hessenberg variety $\mathrm{Hess}(S,h)$. To give a set of ring generators, we used the torus equivariant cohomology and its GKM theory with respect to the natural torus action on $\mathrm{Hess}(S,h)$.

• [Joint work with Naoki Fujita and Haozhi Zeng] We studied the flat family of regular Hessenberg varieties in arbitrary Lie type, and we proved that regular Hessenberg varieties are local complete intersection and that they determine the same cycle in the flag variety. We also showed that higher cohomology groups of structure sheaf of any regular Hessenberg variety vanishes.

• [Joint work with Peter Crooks] We studied a certain family of Hessenberg varieties, and we proved that there is a natural holomorphic Poisson structure with a completely integrable system and that the Toda lattice can be embedded into this variety as a completely integrable systems.

The toric manifolds associated with root systems ([1.9] in the List)

Given a root system Φ, the collection of the Weyl chambers of Φ forms a fan, and we get a non-singular projective toric variety $X(\Phi)$. We provided a combinatorial rule to compute intersection numbers for invariant divisors of $X(\Phi)$ by using Young diagrams. It is known that the cohomology ring of $X(\Phi)$ has a geometric basis, and as a corollary of this rule we provided a recursive formula for the structure constants with respect to this basis.

Schubert calculus for weighted Grassmannians ([1.8], [1.10] in the List)

[Joint work with Tomoo Matsumura] We introduced a natural definition of Schubert classes in the cohomology of the weighted Grassmannian orbifold, and computed the structure constants with respect to them. We also introduced a weighted analogue of Schur polynomials, and studied its relation to geometry and representation theory.