[Further research plan] The objective of this research is **that we make coloring invariants and quantum $U_q(\mathfrak{sl}_2)$ invariants stronger , categorize handlebody-knots and apply them to Prion graphs.** Until now, many invariants of handlebody-knots were separately defined and most of them were coloring invariants, and as for nontrivial quantum invariants, we only had quantum $U_q(\mathfrak{sl}_2)$ invariants. Such handlebody-knot invariants as the following figure 1 and the correlation between them indicate guidelines for a research. The usual knot theory has correlation similar to this figure. We aim at the similar development in case of handlebody-knots to in case of the usual knot theory.

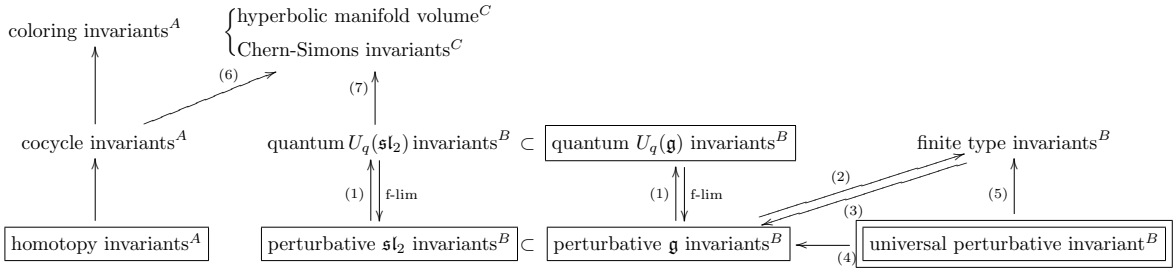

Figure 1: Relation between invariants

Here, invariants without a square are those of handlebody-knots which have already been defined. $\square$ indicates the invariants the applicant will define, and $\boxed{\square}$ does the invariants we will define and research on. Then, a superscript A represents coloring invariants with quandle. Other superscripts B and C represent quantum invariants and hyperbolic geometry invariants respectively. Moreover, an arrow in the figure shows that invariants at the head of the arrow can be gained from those at the tail: invariants at the tail of the arrow are stronger than those at the head. (1)–(7) of the figure 1 show the applicant's conjectures.

As for Prion graphs, our aim is to make them clear by the following operation similar to that of the generic recombination. This could contribute a new idea to the treatment of the Prion Disease.

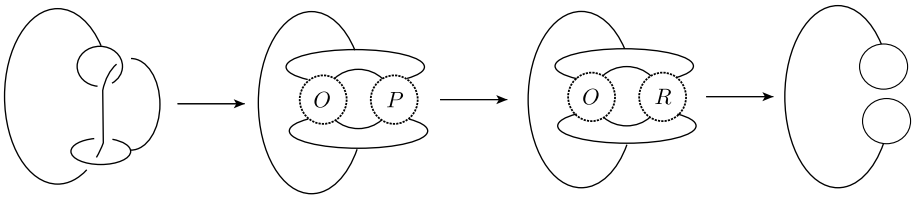

Figure 2: We make a trivial Prion graph by recombining $O + P$ into $O + R$.